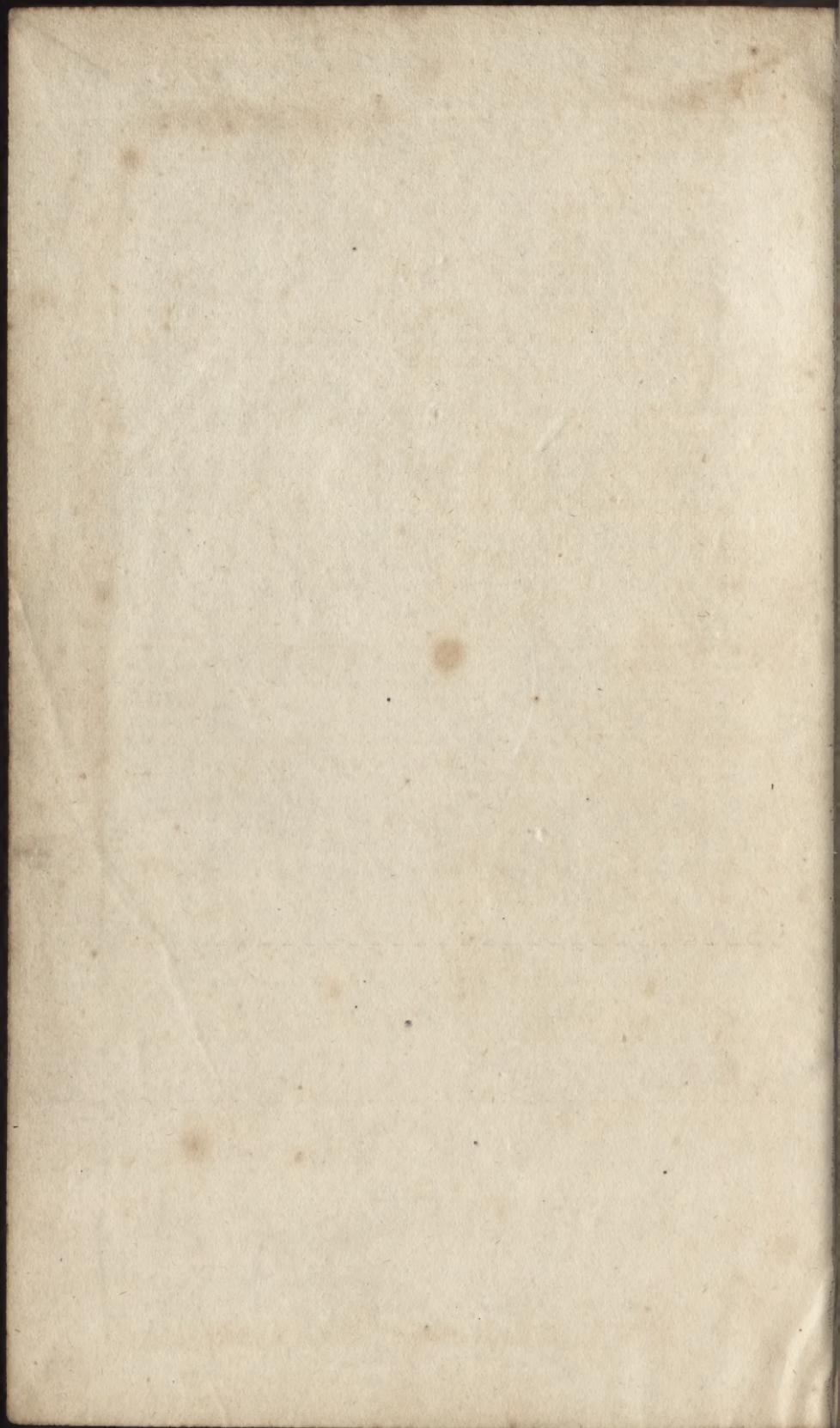
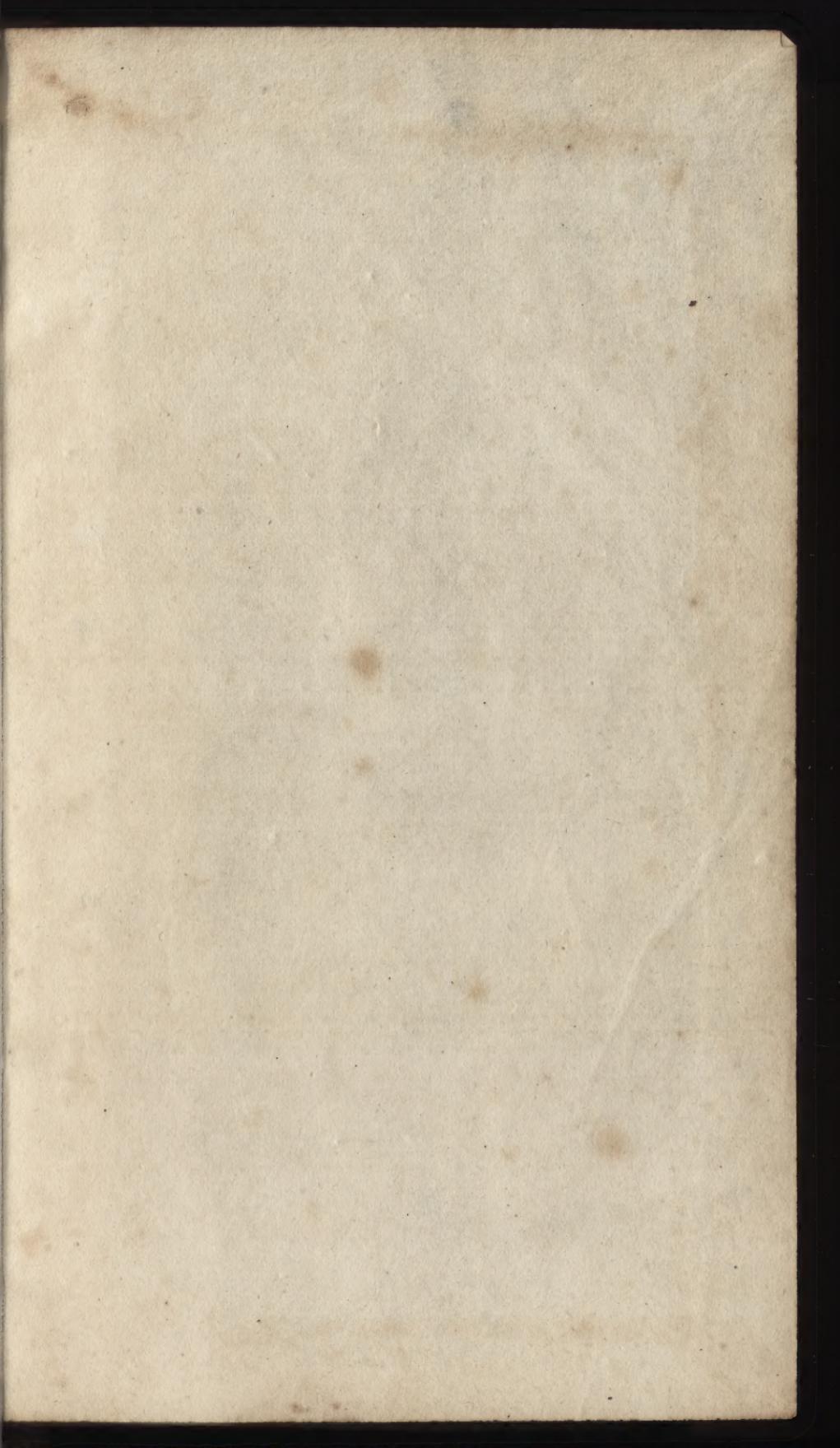






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THE  
PRINCIPLES  
OF  
ARCHITECTURE,

CONTAINING THE  
FUNDAMENTAL RULES OF THE ART,  
IN  
GEOMETRY, ARITHMETIC, & MENSURATION ;  
*With the Application of those Rules to Practice.*

THE TRUE METHOD OF  
Drawing the Ichnography and Orthography of Objects,  
GEOMETRICAL RULES FOR SHADOWS,  
ALSO THE  
FIVE ORDERS OF ARCHITECTURE ;  
WITH A GREAT  
VARIETY OF BEAUTIFUL EXAMPLES,  
SELECTED FROM THE ANTIQUE ;  
AND  
MANY USEFUL AND ELEGANT ORNAMENTS,  
WITH RULES FOR PROJECTING THEM.

---

By P. NICHOLSON, Architect.

Illustrated with Two Hundred and Sixteen Copper-plates, engraved in a  
superior Manner by W. Lowry, from original Drawings by the Author.

IN THREE VOLUMES.

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THE SECOND EDITION, WITH ADDITIONS,  
REVISED AND CORRECTED BY THE AUTHOR.

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VOL. II.

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London :

PRINTED FOR J. BARFIELD, WARDOUR-STREET,  
AND T. GARDINER, PRINCES-STREET, CAVENDISH-SQUARE.

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1809.



## PREFACE.

IN the first volume I have very fully treated on PRACTICAL GEOMETRY. This it is the object of the present volume to apply, in the solution of various useful problems, in the several branches of our art.

I have first shown the method of describing ARCHES of every kind. It is unnecessary to enlarge on the importance of this curious and interesting part of the

subject. To understand it thoroughly, the artist must take care to verse himself well, not only in plain Geometry, but in the properties of the various species of Curves produced by the different Sections of the Cone. I have endeavoured to explain them with as much brevity and perspicuity as possible; and I think the methods I have given are more general, as well as more easy, than those hitherto made public.

I have next explained the manner of describing both Grecian and Roman MOULDINGS, by applying the general principles of the Ellipsis, Parabola, and Hyperbola, to this particular subject. From these curves are produced an infinite variety of the most beautiful figures, from the arrangement of which, with a judicious intermixture of plane surfaces,

surfaces, are derived all that beauty and grandeur, which the various orders of Architecture exhibit.

The methods given by Palladio and Goldman for describing SPIRAL LINES with compasses, are extremely limited; being always confined to three revolutions in the given height, with the eye one eighth part of the whole height. To supply this defect, I have invented a method, easy in its description, and universal in its application; which admits of any number of revolutions whatever in any given height, touching the eye at the end of those revolutions whose diameter is any line given, not exceeding the height of the spiral. These methods may answer very well in describing the volutes of the Roman and Grecian Ionic Order, where they consist

consist of a single fillet, or a fillet and bead; but in describing the Grecian volute on the Temple of Erechtheus at Athens, the proportional spiral only can be properly employed. I have made several attempts to find a method for drawing this curve by a continued motion, but have not hitherto been able to satisfy myself. However I have shown how to obtain any given number of points, through which the curve may be drawn with sufficient accuracy. It may indeed be done accurately enough for common practice by having a point in each quadrant, since the evolute of the proportional spiral is a proportional spiral also, and therefore centres may be found in the evolute, and the spiral drawn with a compass as before.

The ICHNOGRAPHY AND ELEVATION  
OF

OF OBJECTS being necessary to represent their true outline in all the varieties of position to the projecting plane, I have given instructions for them, and then proceed in the last place to treat of the PROJECTION OF SHADOWS: A subject hitherto entirely neglected by writers on Architecture, notwithstanding its importance in orthographical or geometrical designs. Our notions of the effect of any design must certainly be more correct when the drawing from which our judgment is to be formed is a true picture of nature, than when it is shaded merely according to the whim of the artist, without any reference to established rules. The disposition of shadows in nature is not arbitrary. The rays of light are regular in their procession, and it requires only care and attention to

to discover the laws by which the interruption of them is directed and limited. In perspective drawings where we generally see the ceilings and soffits, this is not so important; but in orthographical elevations, we can only judge of the projections by the breadth of the shadows which ought therefore to be exactly proportioned.

P. NICHOLSON.

*October 6, 1808.*

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*End of the Contents of the Second Volume.*



## ARCHES.

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*The Application of Geometry to Architecture,  
in describing elliptical and Gothic Arches,  
and finding the Joints for constructing  
them; also in drawing of Mouldings of  
various Degrees of Curvature, &c.*

---

### DEFINITIONS.

1. **A** N Arch, in architecture, is a number of stones, or any other mass of matter, capable of being built over a hollow space, in the form of some curve; such as a semicircle, a semiellipsis, a parabola, an hyperbola, or a catenarian, &c. or any part of these curves.

2. A simple arch is a continuation of the same curve throughout the whole; such as a segment of a circle, or a segment of an ellipsis, a parabola, &c.

3. A compound arch is that which is not a continuation of the same curve, as is generated by a continued motion: thus, two segments of a circle, of a different radius, joined together, touching a straight line at the point of their junction, is a compound arch.

4. A gothic arch, in general, is formed of two segments of the same species of curve, meeting in a point at the top, called the vertex, and whose extremities in general touch parallel planes, perpendicular to the horizon.

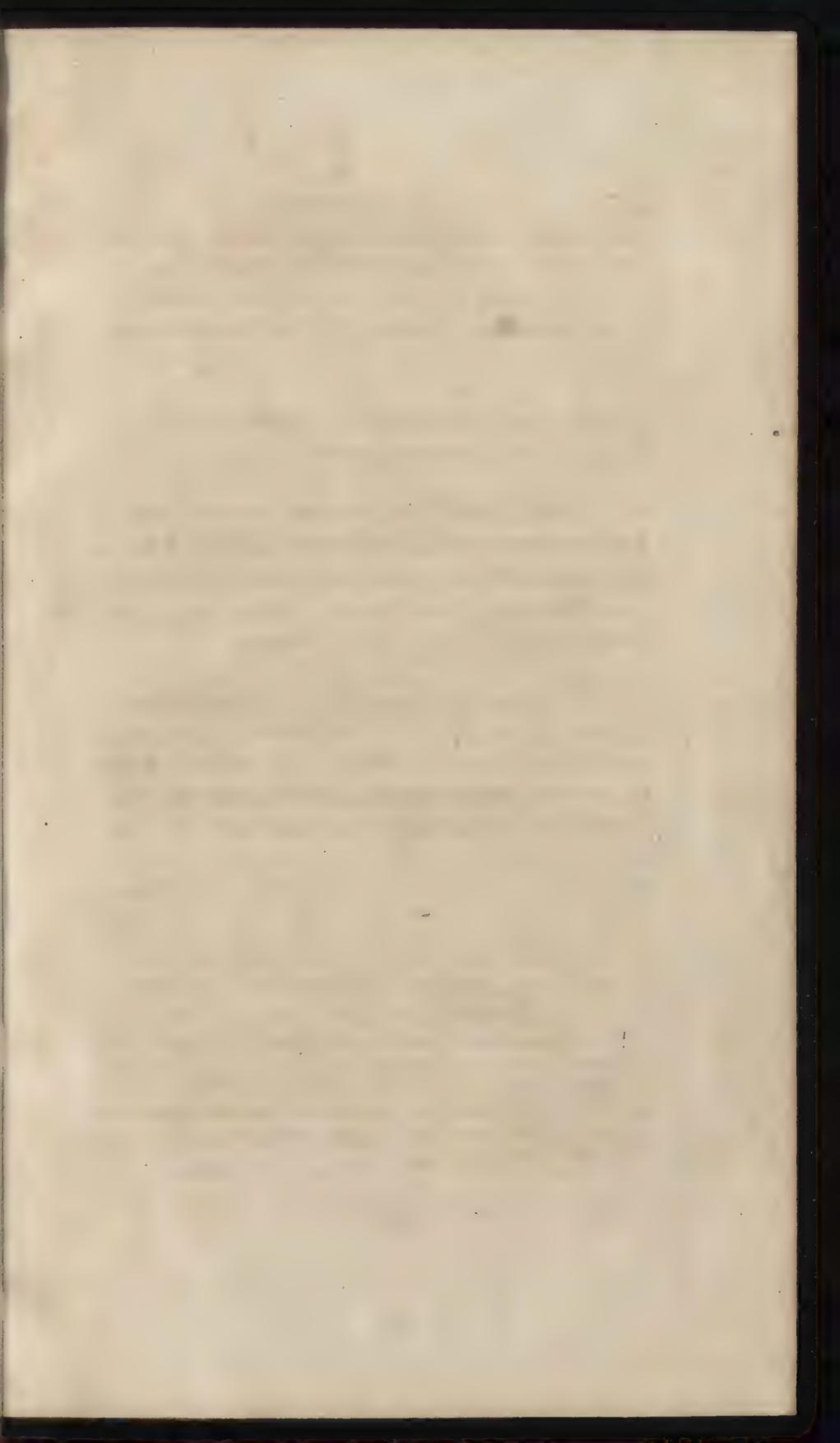
Gothic arches may be made of two segments of a circle, or two segments of an ellipsis, &c.

5. A right arch is such, that the extremities spring from two planes, which are tangents to the arch at the intersection of a given horizontal plane, and the two tangential planes being equally inclined to vertical planes passing through their intersections.

6. An oblique or rampant arch is such, that the extremities spring from the intersections of two planes, equally inclined to the horizon, which are tangents to the arch at their intersection with another plane, inclined in any given angle to the horizon.\*

#### PLATE

\* The author having extended the plan of this work much beyond his original intention, and he hopes greatly to the student's advantage; in consequence of which, all the plates in this edition are numbered in regular succession, in order that the subjects of the work may follow each other with precision.



Pl. 59.

Fig. 1.

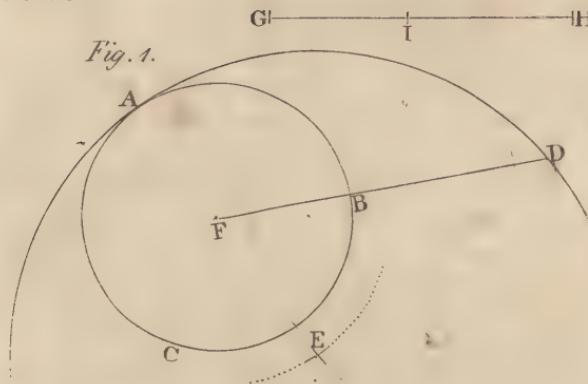


Fig. 2.

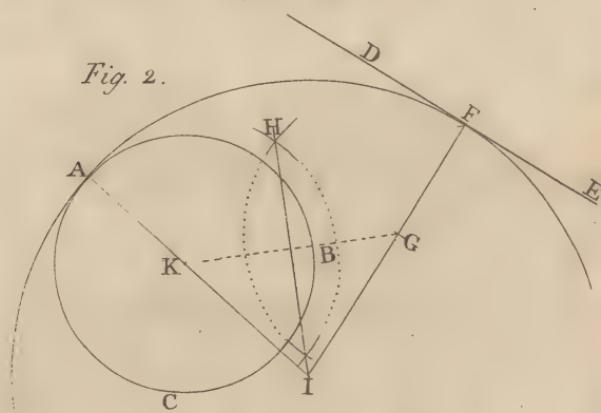
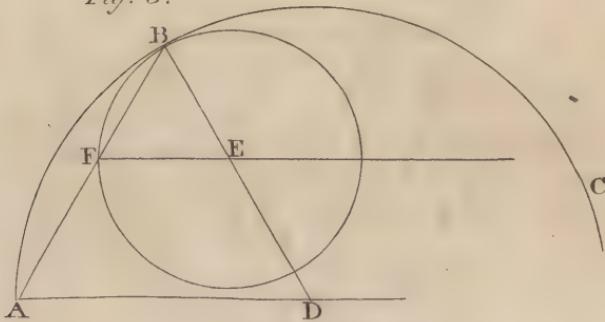


Fig. 3.



## PLATE 59. FIG. 1.

*To describe the circumference of a given circle G H through a given point D, to touch a given circle A B C, whose centre is F; provided that the sum of the difference between the two given radiusses, and the radius of the required circle, shall exceed a straight line, drawn from the given point to the centre of the circle given in position.*

Join F D; from G H, cut off the part G I, equal to the radius of the circle A B C, and with the difference I H, on the centre F, describe an arc at E; on D, as a centre with a radius G H, describe an arc cutting the former at E; lastly, on E, as a centre with the distance E D, describe a circle, it will touch a circle A B C, as was required.

## FIG. 2.

*To describe a circle, to touch a given circle A B C, whose centre is K, and to touch a given straight line D E, in the point F.*

From F, draw F I, perpendicular to D E; from the point F, and from the line F I, cut off the part F G, equal to the radius of the circle A B C; join K G, and bisect it by a perpendicular, cutting F I, at I; through the points I and K, draw the straight line I K A; lastly, with the radius I A, or I F, describe a circle and it is done.

## FIG. 3.

*To describe a circle, to touch a given circle A B C, whose centre shall be in a given straight line F E, and pass through a given point F in that line.*

Through D, the centre of the given circle A B C, draw D A, parallel to F E, cutting the circumference of the circle A B C at A; through the points A and F, draw the straight line A F B, cutting the circumference of the circle A B C at B; draw the straight line B E D, cutting F E at E; then on E, as a centre with the distance E B, or E F, describe a circle, and it is done.

## PLATE 60. FIG. 1, 2, and 3.

*To describe a parabola by means of tangents having a double ordinate A B, and a diameter E D to that double ordinate.*

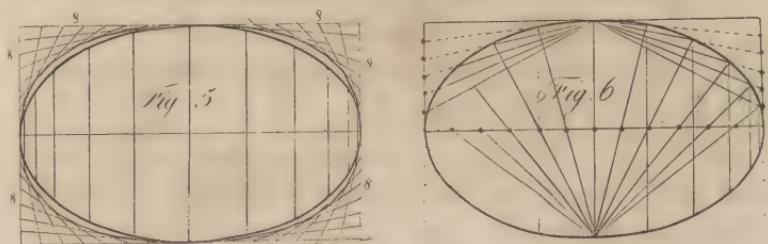
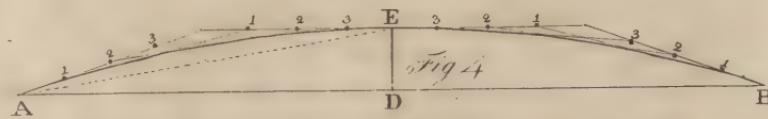
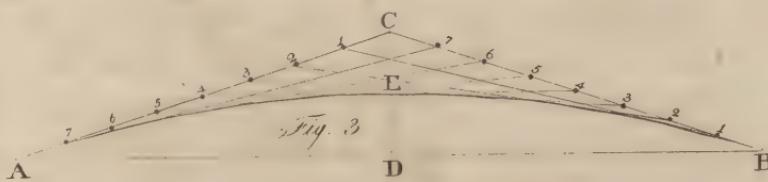
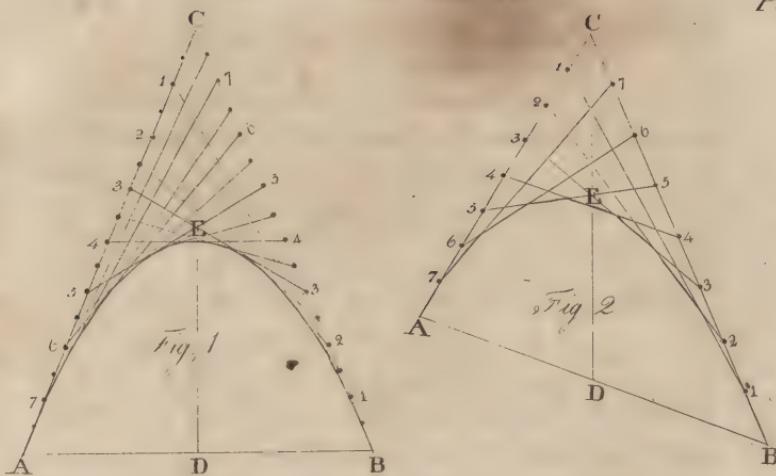
Produce the diameter D E towards C; make E C equal to E D; join A C and B C, divide each of the lines A C and B C into a like number of equal parts, and join the corresponding points of the divisions, and it is done.

Although any mathematical reader may easily prove that a figure described in this manner is a parabola; yet such is the ignorance of many architects, and of workmen in general, that they pretend they can by this method describe a figure almost of any form they please.

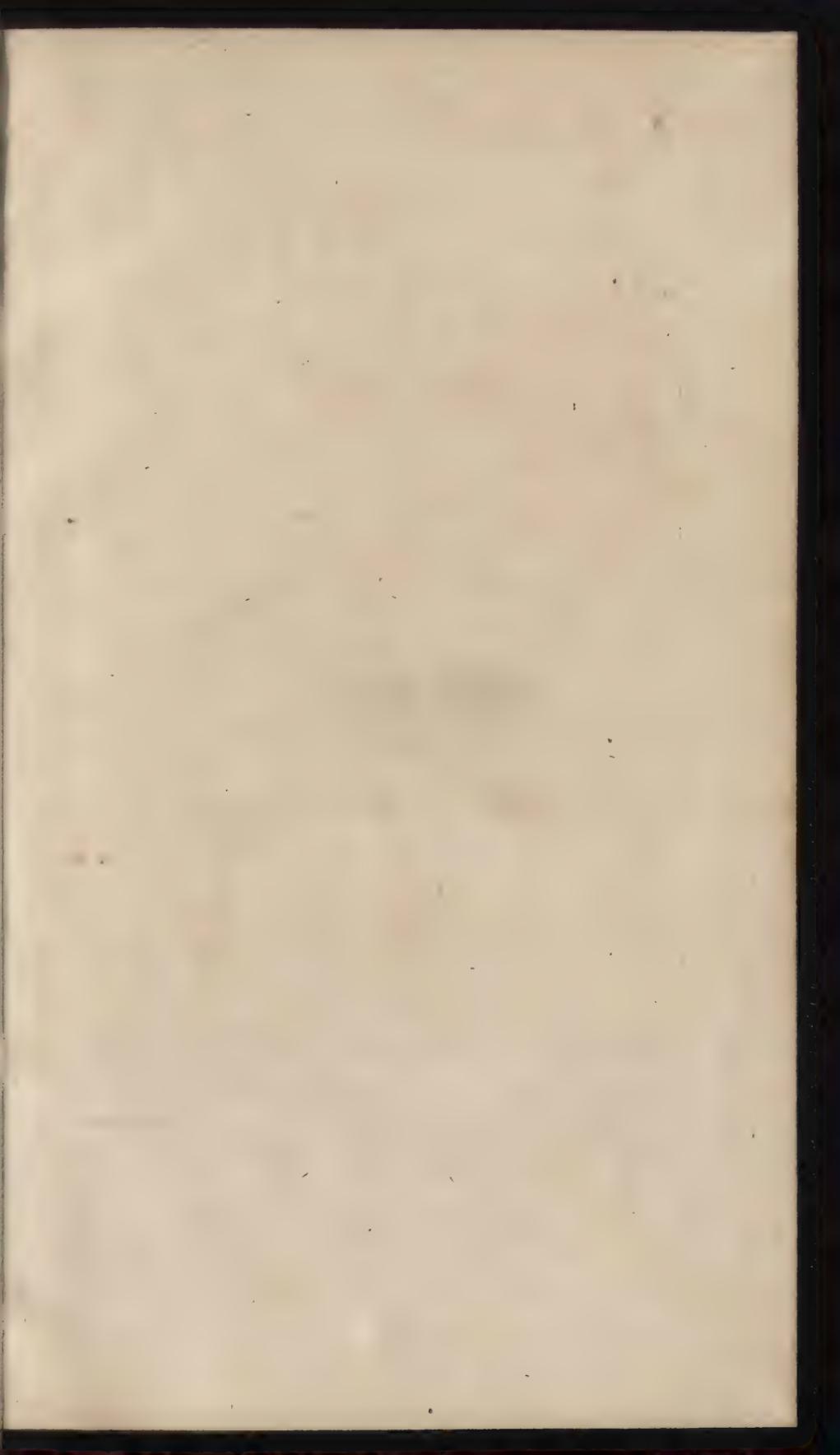
If the parabola be very flat, as in Fig. 3, it may be used in practice, without any sensible error, for the segment of a circle; or it may be described as in Fig. 4. by describing each half of the figure as before; it has also been applied in describing an ellipsis, as is shown at Fig. 5; but the difference of a figure compounded of parabolas from the true ellipsis, is very visible to every discerning eye, as is shown in Fig. 5, where the inside curve is an ellipsis taken from Fig. 6; the parabolic curve seems more particularly adapted to the description of gothic arches, as applied in some of the following examples.

Fig. 7, is another ridiculous application of parabolic curves, in order to produce the form of an egg, or oval; but to an eye that has been accustomed to curves, generated by a continued motion of an instrument, this curve is very ungraceful.

Fig. 8, shows a method, by means of sines, of inscribing an ellipsis within a trapezoid.







Pl. 61.

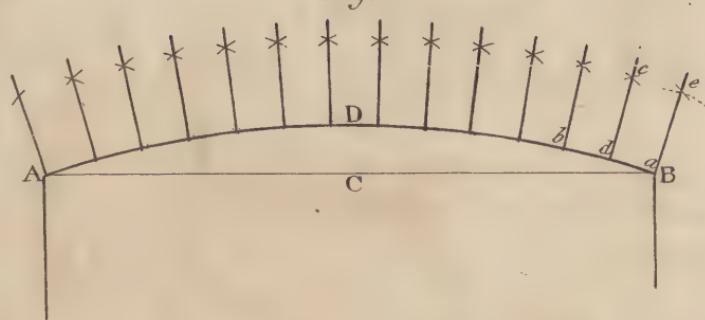


Fig. 1.

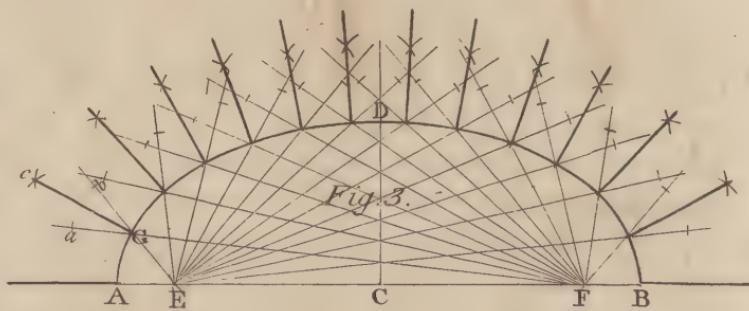
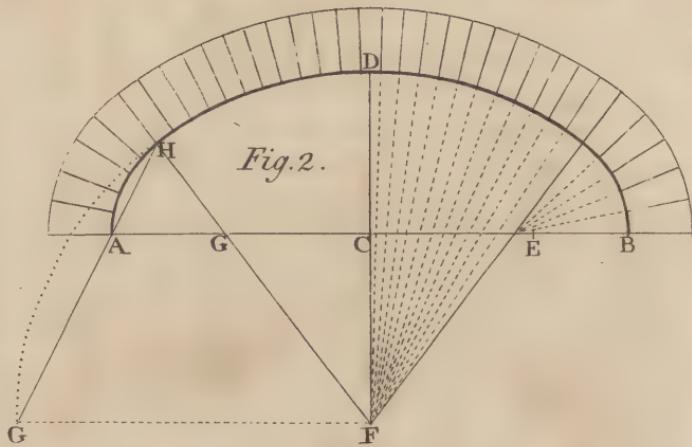


Fig. 3.

## PLATE 61. FIG. 1.

*To find the joints of an arch A B D, that is the segment of a circle.*

Divide the arch into as many equal parts as you intend to have joints: to make a joint at any point  $d$ , that is not at one of the extremes of the arch, take any two equal distant points  $a$  and  $b$ , on each side of the point  $d$ ; and with any radius greater than  $b d$ , or  $d a$ , on the points  $b$  and  $a$ , as centres, describe two arcs, cutting each other at  $c$ ; draw  $c d$ , and it will be one of the joints at the point  $d$ ; in the same manner find all the other joints except the two extremes, which may be found as follows; suppose it were required to find the extreme joint at  $B$ , take any distance  $d c$ , on any one of the joints that is found, and with that radius on  $B$ , describe an arc at  $e$ ; and with the distance  $B C$ , on the point  $d$  describe an arc cutting the former, and draw  $E B$ , and it will be the joint required.

## FIG. 3.

*To find the joints of an elliptic arch A B D.*

Find the foci  $E$  and  $F$ , by Prob. 1, Page 22; then, to draw a joint through any point  $G$ , in the arch, draw the straight lines  $F G a$  and  $E G b$ ; and bisect the angle  $a G b$  by the line  $c G$ , and it will be a joint at the point  $G$ : in the same manner all the other joints may be found.

Fig. 2, is an arch made of the sectors of circles, representing an elliptical arch, the joints in any of the sectors will be found, by drawing them to their corresponding centres.

## PLATE 62. FIG. 1.

*To draw a gothic arch of any height C D, and width A B, and to touch two given lines D G and D H, making equal angles with C D.*

Draw A G and B H perpendicular to A B, cutting D G and D H, at G and H; join G H, cutting C D at E; then apply the distance E D, from C to F, in the same straight line; join F B and F A; divide each of the lines F B, and F A, A G and B H into a like number of equal parts, draw lines through the points 1, 2, 3, 4, 5, in A G and B H to the vertex D; also draw lines through the points 1, 2, 3, 4, 5, in the lines A F and B F to I, cutting the former, as is shown in the figure, and these will be the points in the curve.

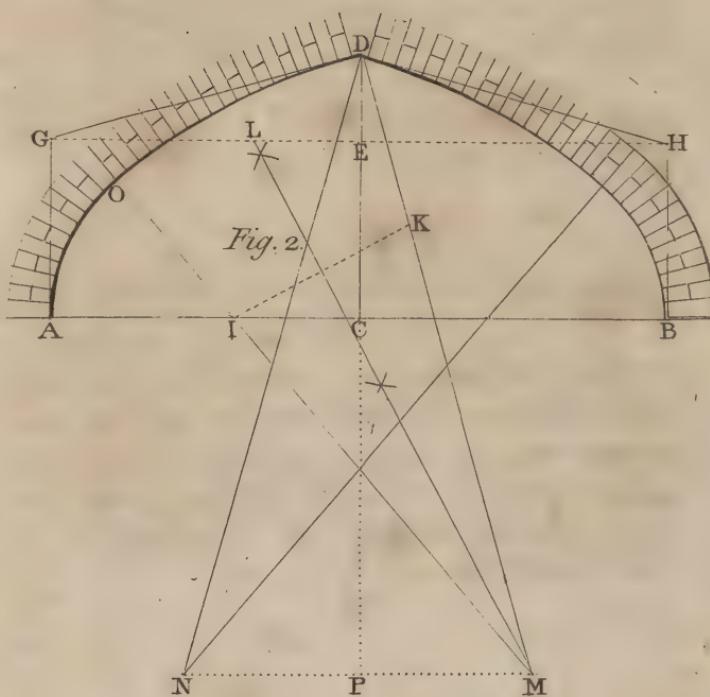
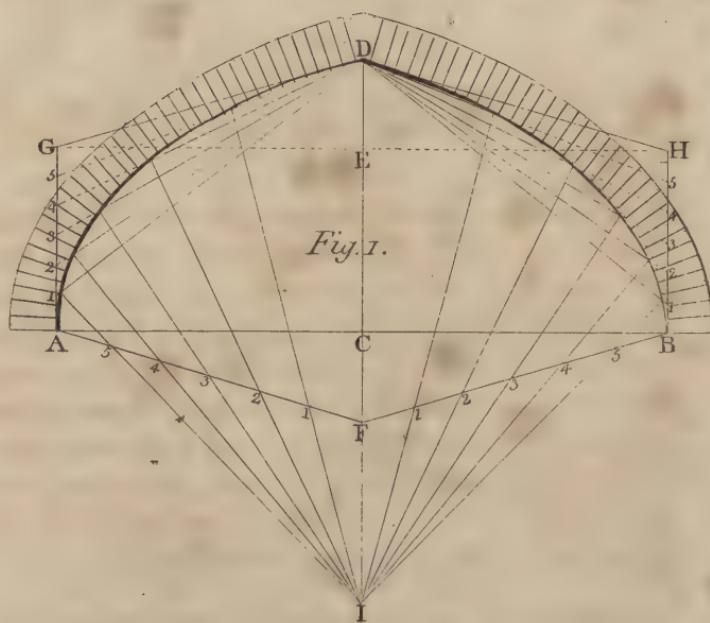
If the point E be distant from C, more than one half of C D, the two sides of the gothic arch will be two parts of an ellipsis; but if the point E falls in the middle of C D, then the two sides of the arch are parabolical; lastly, if E fall under one half of C D, then each side of the arch is hyperbolical.

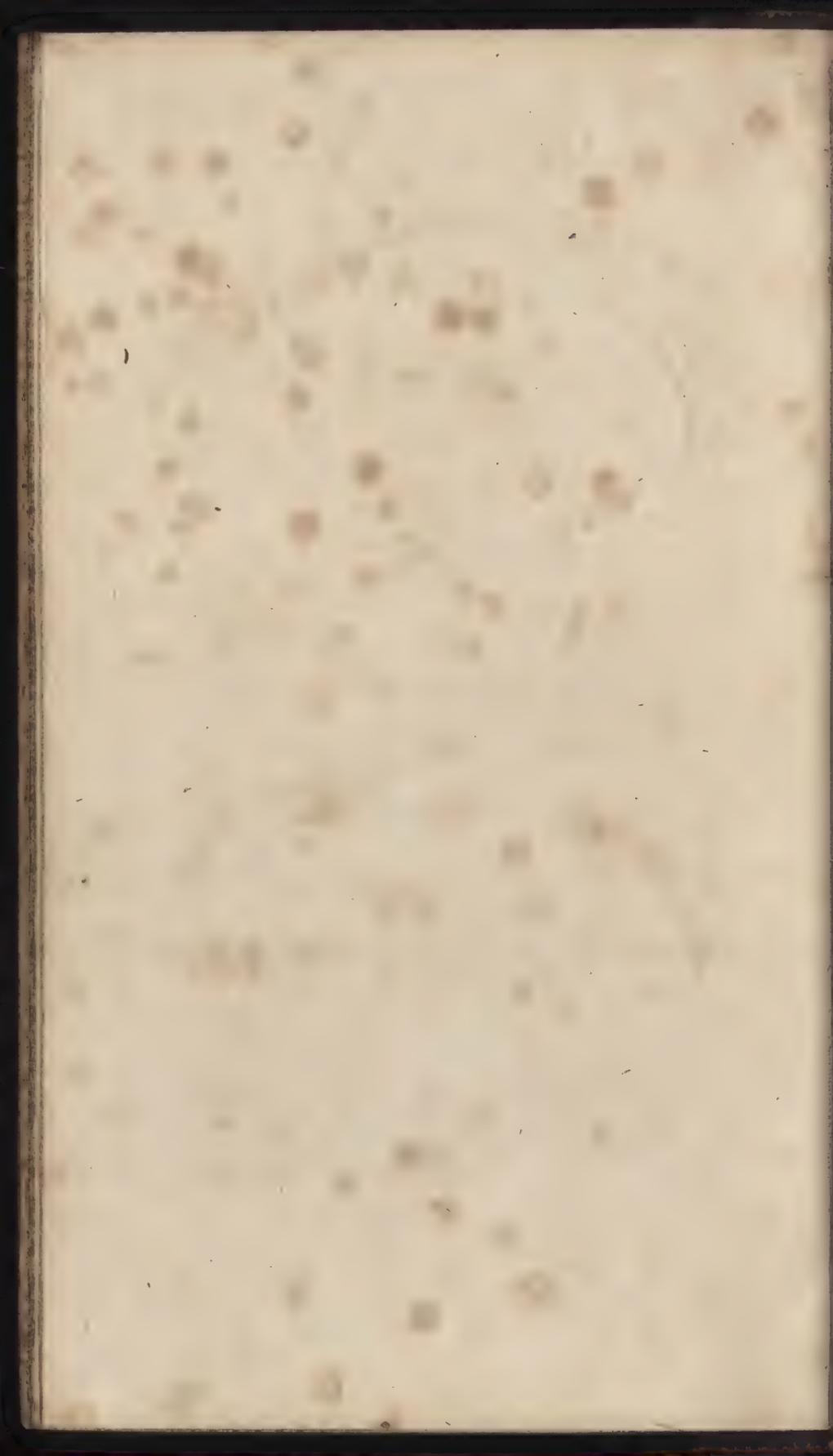
## FIG. 2.

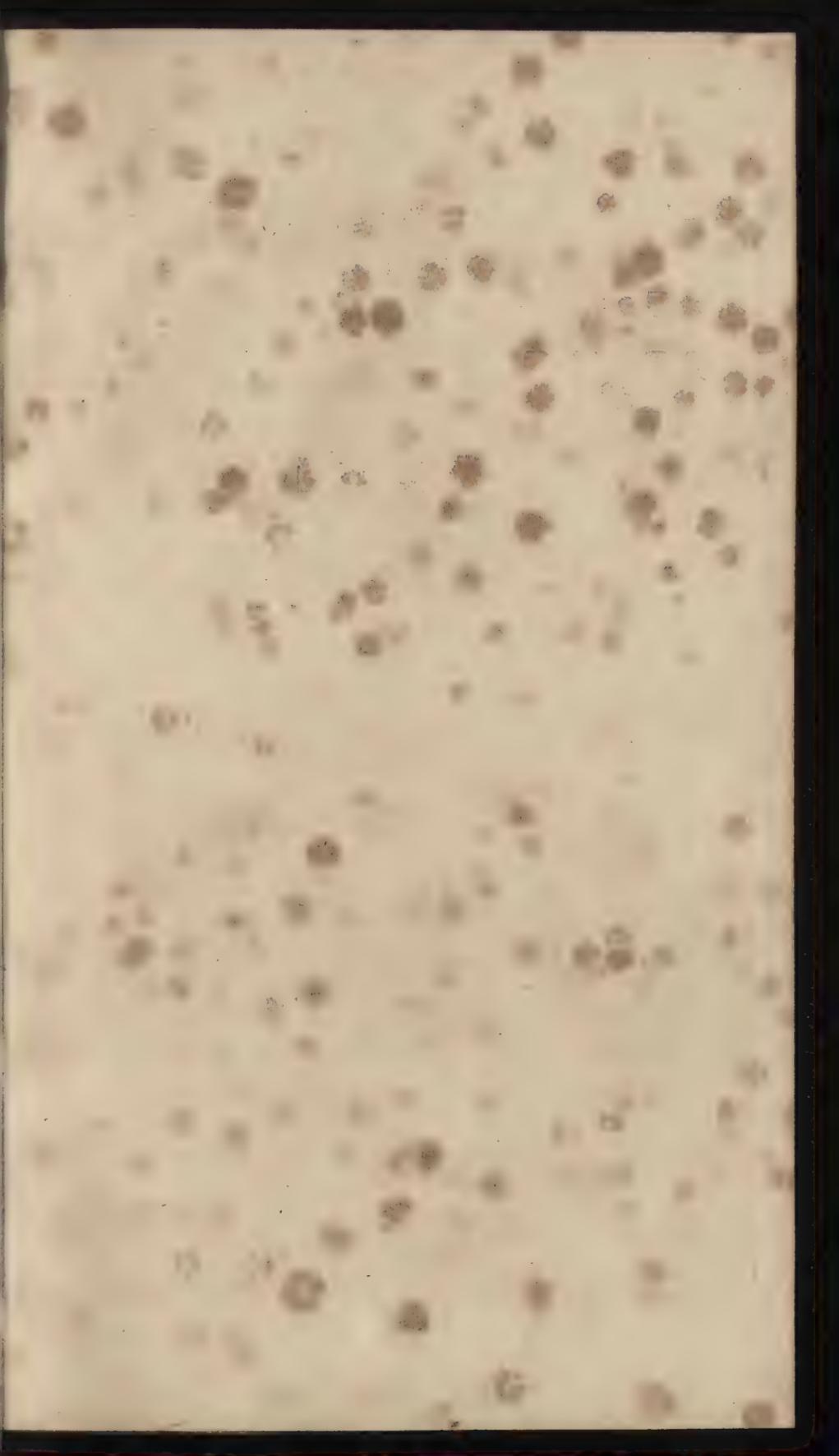
*The same things being given, to describe the representation of such an arch with a compass.*

Make A I equal to A G; and describe on I, with the radius I A, a part of a circle A O L; then describe a circle to touch the straight line D G, at the point D, and the circle A O L, and it will complete one side of the arch, the other side may be completed in the same manner, as is plain by the figure.

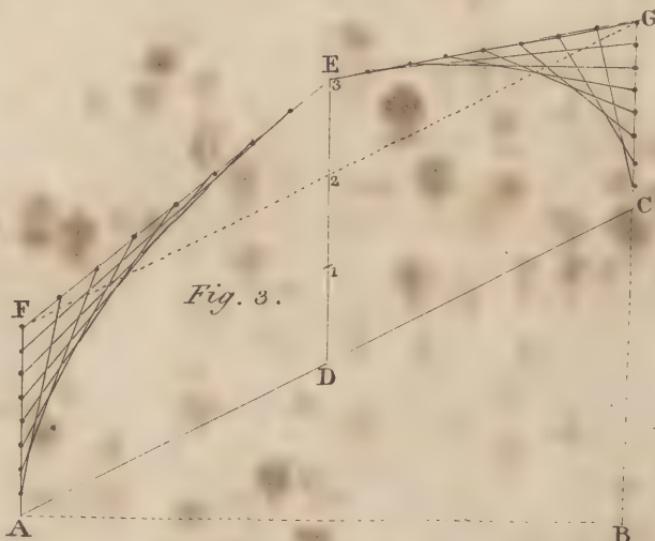
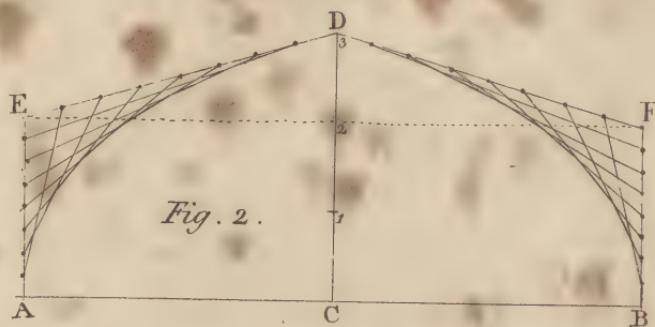
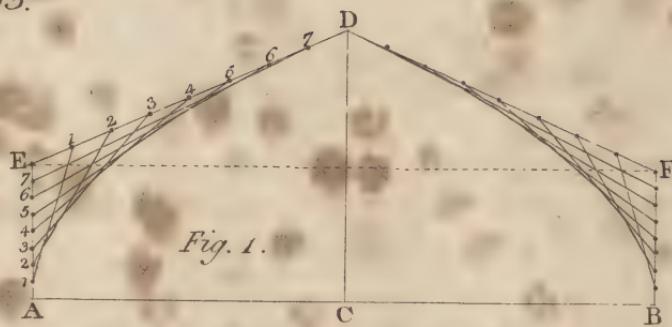
## PLATE







Pl. 63.



## PLATE 63. FIG. 1, 2, and 3.

*To describe parabolical gothic arches, whether right or rampant, to any two lines meeting at the vertex; and also to touch any two lines from the ends of the arch.*

Proceed with each half of the arch, as in plate 91, Page 4; that is, describe a parabola on each side, and it is done.

## PLATE 64. FIG. 1.

*To describe a gothic arch by finding points in the curves, the tangent D E and D F, at the vertex of the arch being given.*

Join D A and D B, and bisect them at the points G and H; join also E G and H F; bisect them at the points K and I; then will the lines K G and H I, be diameters; D A and D B will be a double ordinate corresponding to each; (then by Prob. 1, Page 34, Geometry) describe each parabola, and they will form the gothic arch required.

## FIG. 2.

*To find the joints of a gothic arch described as above.*

Suppose it was required to draw a joint to a given point  $a$ ; through  $a$ , draw  $a c$ , parallel to A D, cutting the diameter E G at  $c$ ; from the point  $d$ , where E G cuts the arch, make  $d e$  equal to  $d c$ ; join  $e a$ , and it will be a tangent at the point  $a$ ; a line drawn through  $a$ , perpendicular to  $a e$ , will be the joint sought; in the same manner all the other joints are to be found.

The joints are to be found in Fig. 3, in the same manner.

MOULDINGS

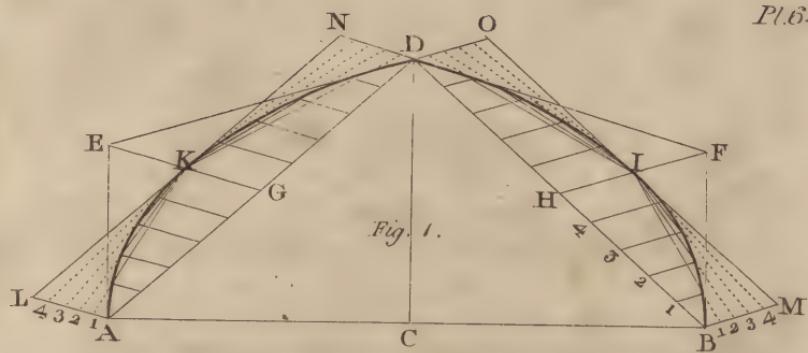


Fig. 1.

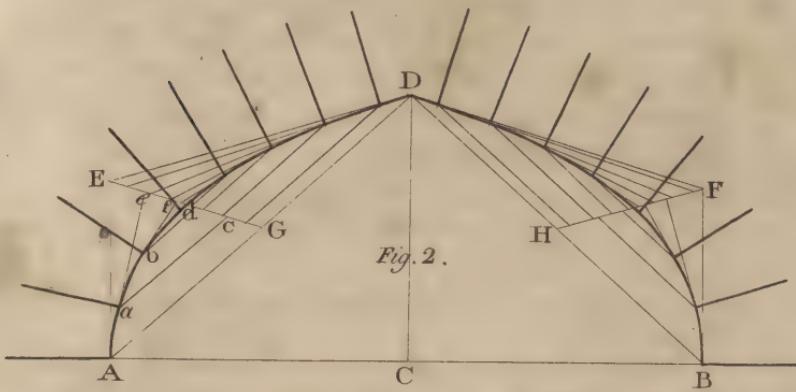


Fig. 2.

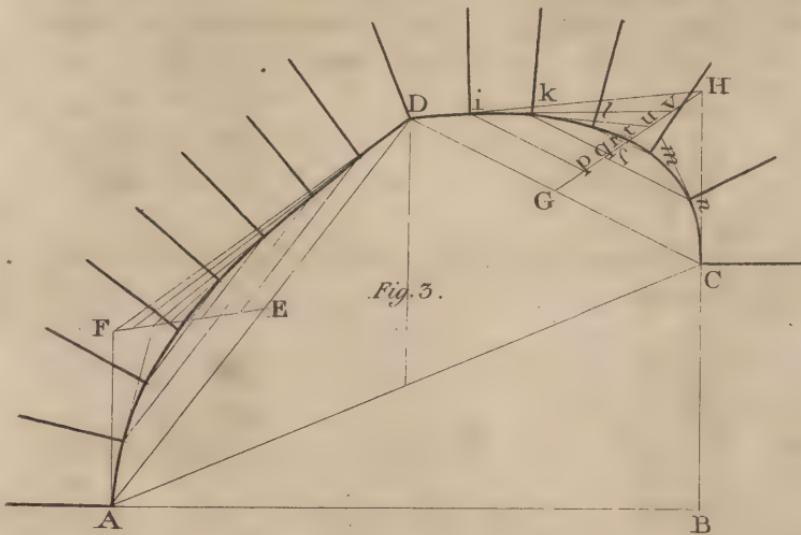
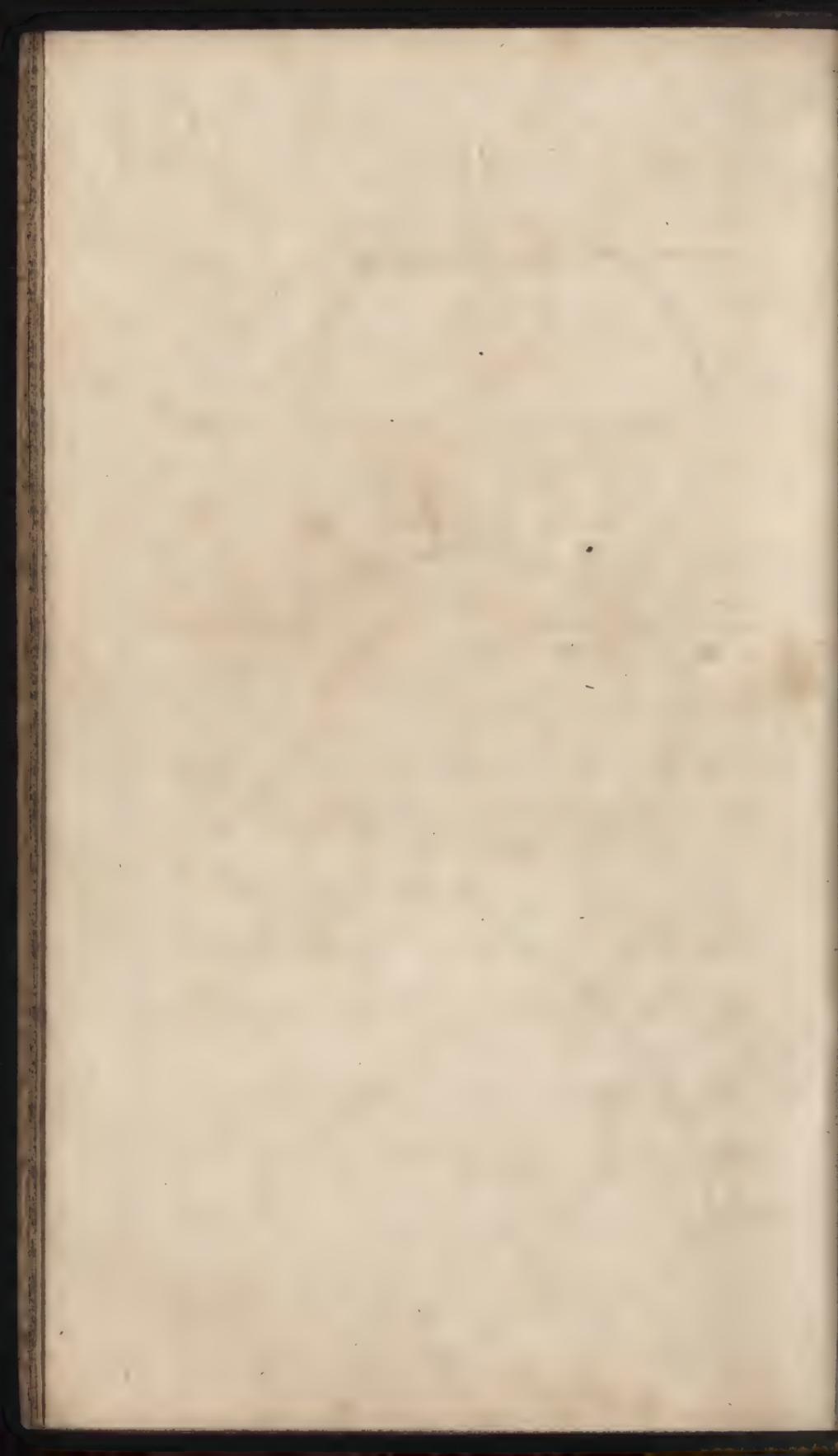


Fig. 3.



## MOULDINGS.

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### DEFINITIONS.

1. Mouldings are figures composed of various curves and straight lines.

If the mouldings are only composed of parts of a circle, and straight lines, they are called Roman, because the Romans, in their buildings, seldom or never employed any other curve for mouldings than that of a circle ; but if a moulding be made of part of an ellipsis, or a parabola, or an hyperbola, the mouldings are then in the Grecian taste.

*Corollary.* Hence it appears, that mouldings in the Greek taste are of a much greater variety than those of the Roman, where only parts of circles are concerned.

Mouldings have various names, according to the manner in which they are curved.

2. The straight lined part under or above a moulding in general, is called a fillet.

3. If the contour of the moulding be convex, and a part of a circle equal to, or less than a quadrant, then the moulding is called a Roman ovolo, or an echinus, such as Fig. 2, Plate 66.

4. If the contour of the moulding be concave, and equal to or less than a quadrant, it is called a cavetto, or hollow, such as Fig. 3, Plate 66.

*Corollary.* Hence a cavetto is just the reverse of an ovolo.

5. A bead is a moulding, whose contour is simply a convex semicircle.

6. If the contour be convex and a complete semicircle, or a semiellipsis, having a fillet above or below it, the moulding is called a torus, as Fig. 1, Plate 66.

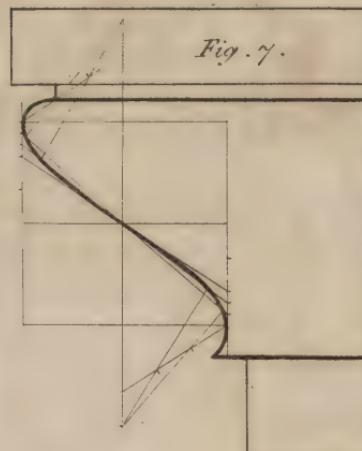
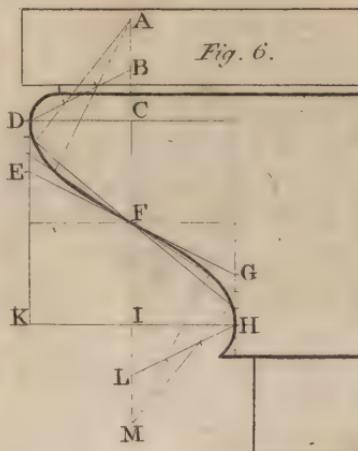
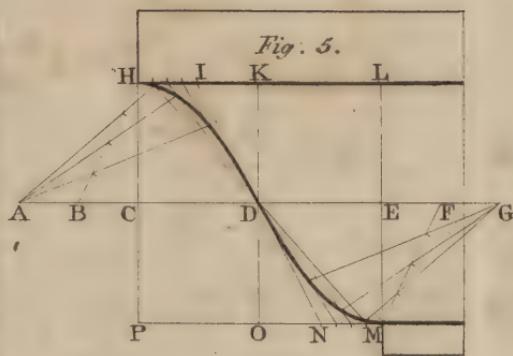
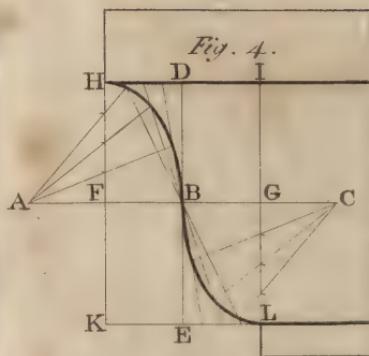
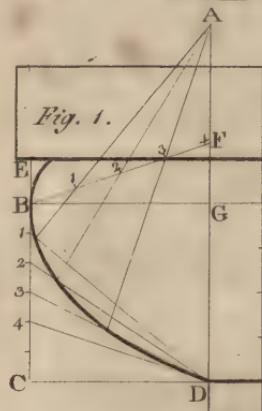
*Corollary.* Hence a torus is a bead with a fillet, and is more particularly distinguished in an assemblage of mouldings from a bead, by its convex part being much greater.

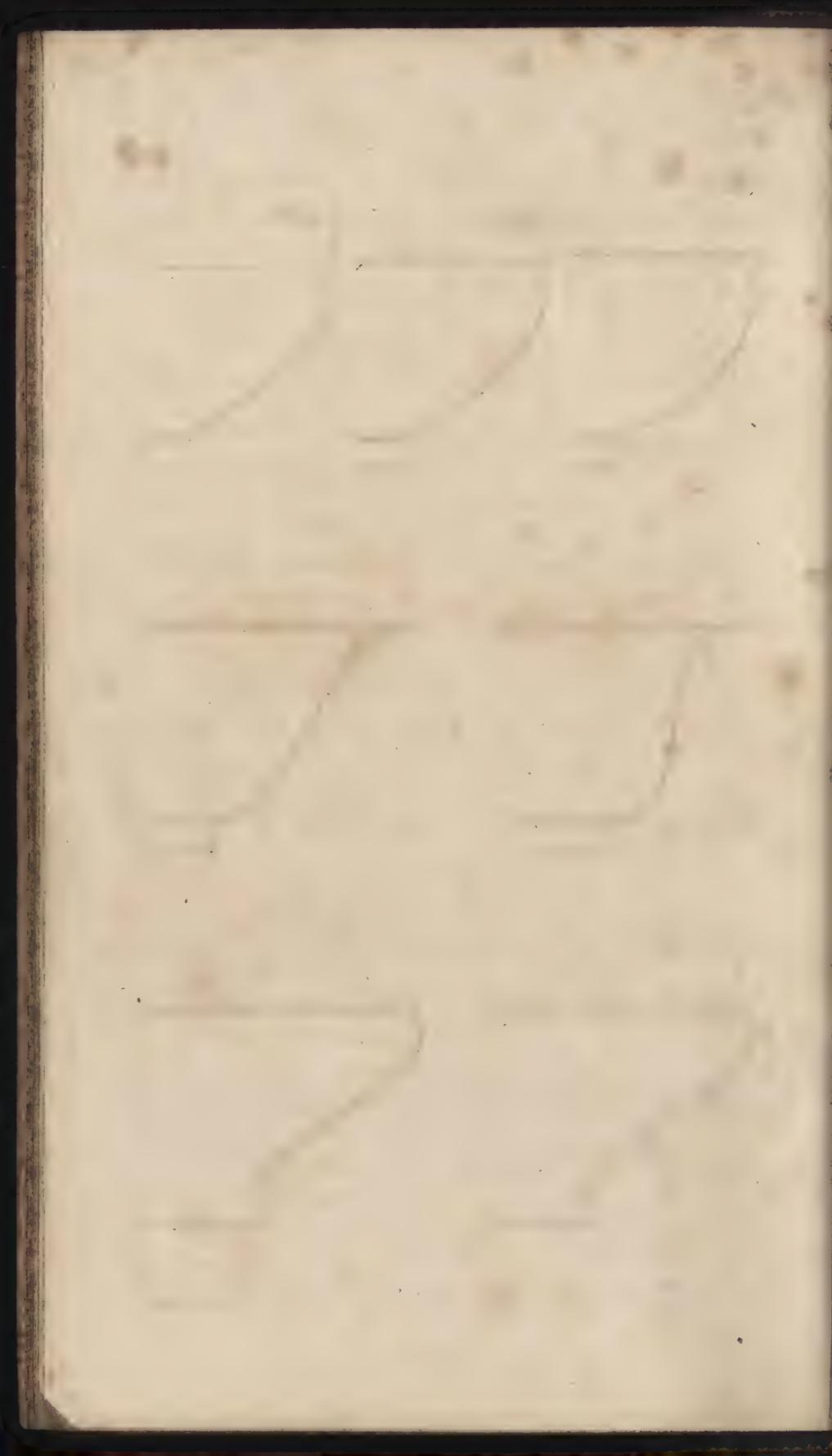
7. If the contour of a moulding be a concave semiellipsis, it is called a scotia, as Fig. 6, Plate 68.

8. If the contour be convex, and not made of any part of a circle, but of some other of the conic sections, having a small bending inwards towards the top, the moulding is called a Grecian ovolo, or echinus, such as Fig. 1, 2, 3, 4, 5 and 6, Plate 67.

9. If the contour be partly concave, and partly convex, the moulding in general is called a cimatum, such as Fig. 4, 5, 6 and 7, Plate 65; also Fig. 5 and 6, Plate 66.

## GRECIAN MOULDINGS.





10. If the concave parts of the curve project beyond the convex part, the cimatum is called a *cimarecta*; such as Fig. 4, Plate 65.

11. If the convex part project beyond the concave, the cimatum is called a *cimareversa*, or *ogee*.

12. The bending, or turning inwards, of a small part of the convex curve of a Grecian moulding, is, by workmen, called a *quirk*.

## PLATE 66. FIG 1.

*To describe a torus.*

Bisect the diameter at  $a$ ; on it with the radius, describe a semicircle, and it is done.

## FIG. 2.

*To describe an ovoli in the Roman taste; the projections at a and b being given at each extreme of the curve.*

Take the height of the moulding; on the points  $a$  and  $b$  as centres, describe an arc at  $c$ ; on  $c$ , as a centre with a radius  $c a$  or  $c b$ , describe the arc  $a b$ , and it will be the contour of the moulding required.

## FIG. 3.

*To describe a cavetto, having the extremes of the curve.*

The cavetto is described in the same manner, but on the opposite side.

## FIG. 4.

*To describe a hollow, to touch with two straight lines b d and d a, one of them at a given point a.*

Let  $d$  be the point of their meeting; make  $d b$  on the other line, equal to  $d a$ ; from the points  $a$  and  $b$ , draw perpendiculars to each of the lines  $d b$  and  $d a$ , meeting at  $c$ ; on  $c$ , as a centre, with the radius  $c b$ , or  $c a$ , describe an arc  $b a$ , and it is done.

## FIG. 5.

*To describe a cimarecta, the projections at a and b being given.*

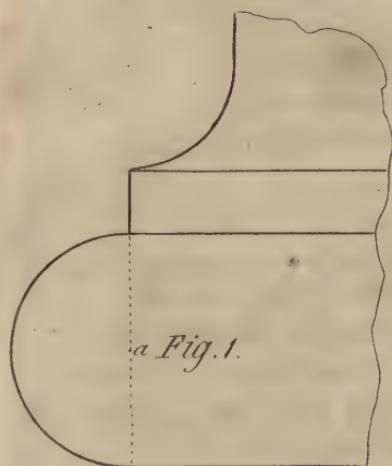
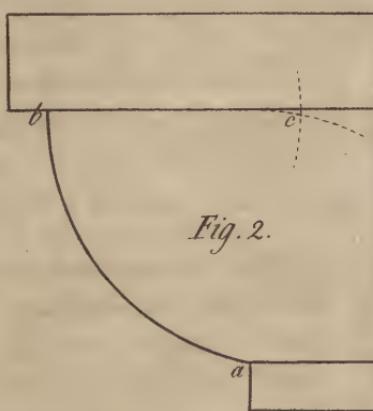
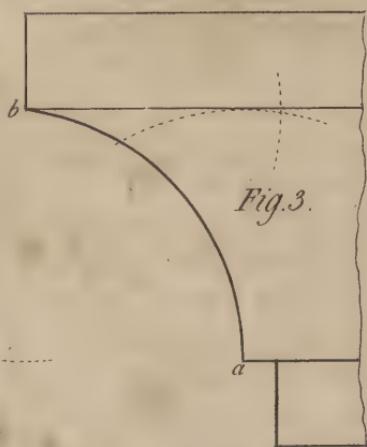
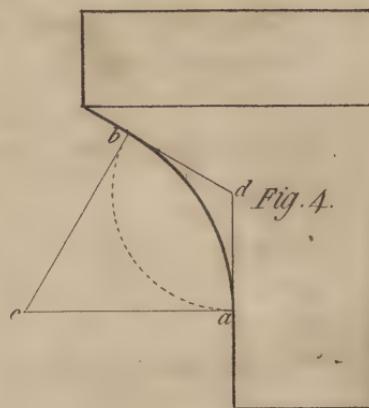
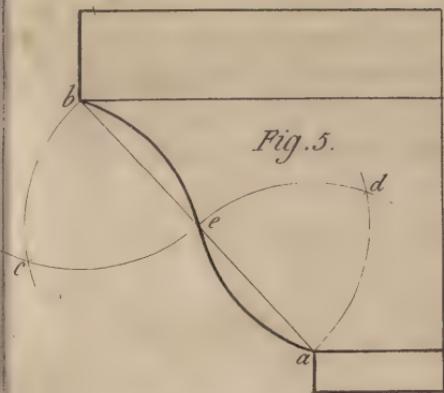
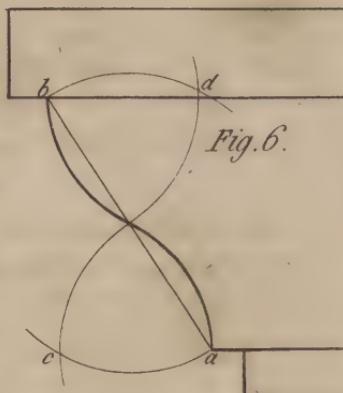
Join  $a b$ ; bisect it at  $e$ ; then on the points  $a$  and  $b$ , describe arcs meeting each other on the opposite sides at  $c$  and  $d$ : on the points  $c$  and  $d$ ; with the same radius, describe the opposite curves  $a e$  and  $e d$ , and it is done.

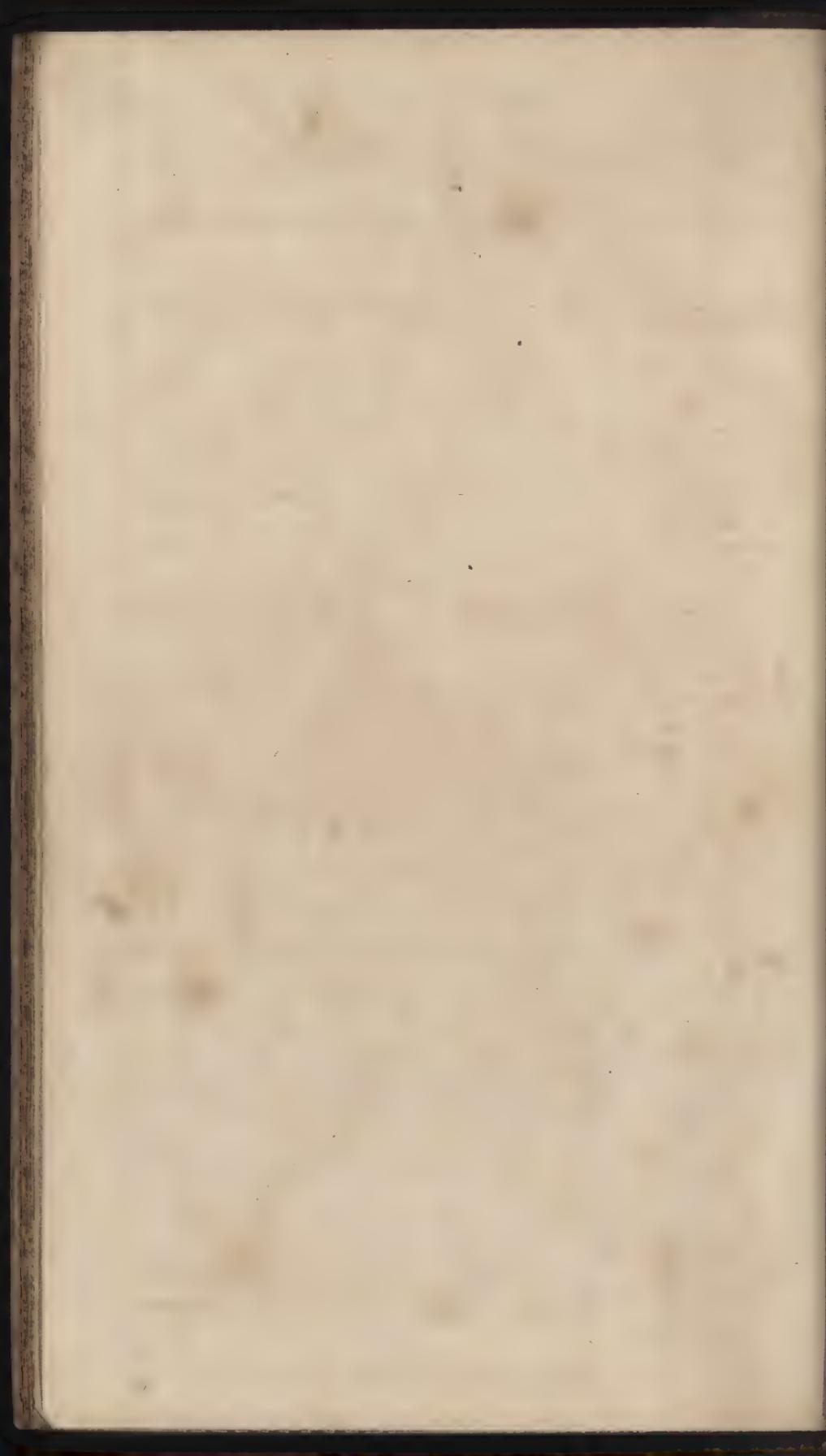
## FIG. 6.

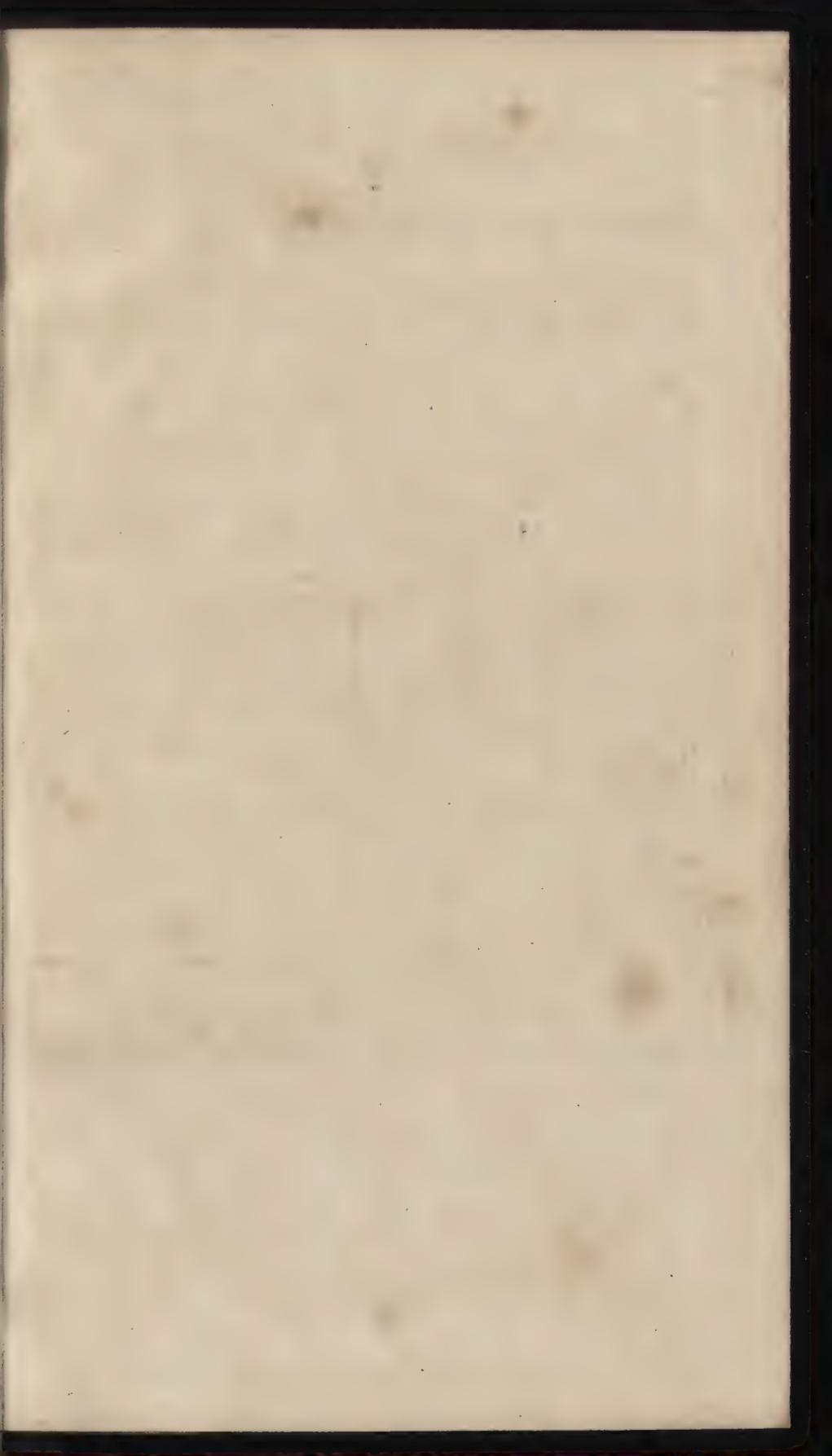
The cimareversa is described in the same manner, but in an opposite direction.

**ELLIPTIC**

## ROMAN MOULDINGS

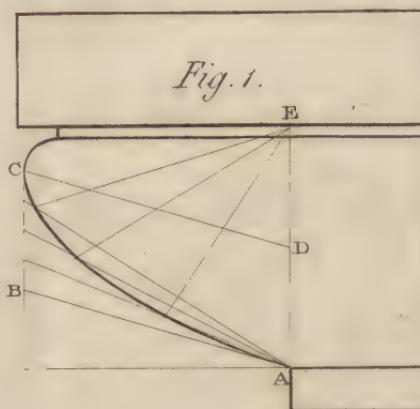
*Fig. 1.**Fig. 2.**Fig. 3.**Fig. 4.**Fig. 5.**Fig. 6.*



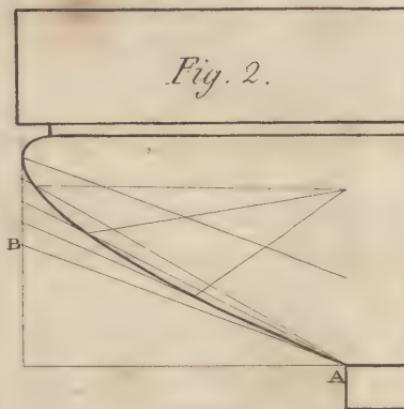


GRECIAN MOULDINGS

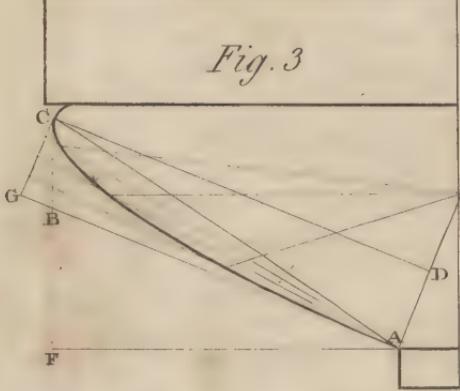
*Fig. 1.*



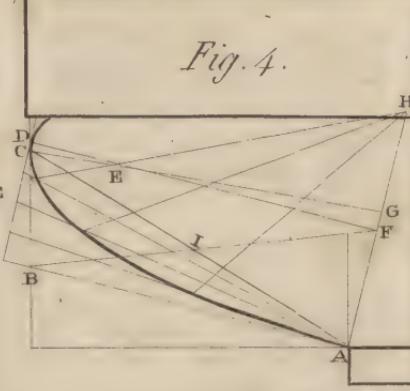
*Fig. 2.*



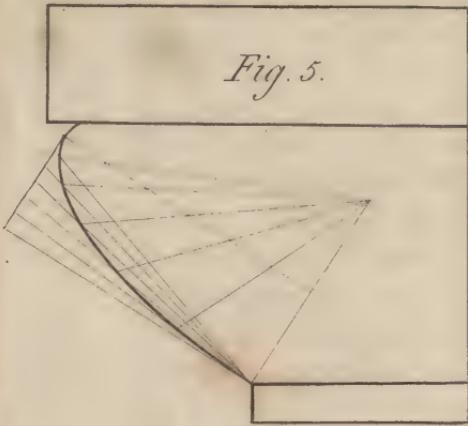
*Fig. 3*



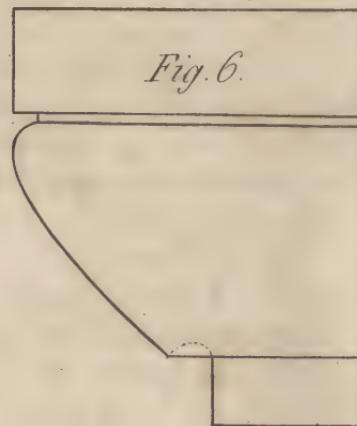
*Fig. 4.*



*Fig. 5.*



*Fig. 6.*



*P.Nicholson del.*

*Published Oct<sup>r</sup>. 1. 1795. by P.Nicholson & C<sup>o</sup>.*

## ELLIPTIC MOULDINGS.

PLATE 67. FIG. 1.

To describe the Grecian echinus, or ovoli; the tangent A B at the bottom, the point of contact A, and the greatest projection of the moulding at C, being given.

From A, draw A D E perpendicular; through C draw C B parallel to it; also through C draw C D, parallel to the tangent B A, cutting A E at D; make D E equal to A D; then will D be the centre of an ellipse, and C D and D A will be two semiconjugate diameters; from which the ellipse may be described, by Prob. 6, Page 24, Geometry.

FIG. 2.

This figure is described in the same manner, and shows a greater projection; the tangent being also taken in a higher position.

FIG. 3.

The same things being given, to describe the moulding nearly, when the point of contact A is at the extremity of the transverse axis.

From A draw A D E perpendicular to the tangent B A; parallel to it, draw C G, cutting the tangent A B at G; also through C draw C D parallel to the tangent A B, cutting A D E at D, the centre of the ellipse for which D C and D A are the semitransverse and semiconjugate axes; and proceed as before.

FIG. 4.

To do the same exactly true, having the same things given.

Draw A F H perpendicular to B A, as before, join C A and bisect it; from the point B, where the two tangents C B and A B meet, draw B F through the middle of A C, cutting A H at F; through F draw F D, parallel to A B; on C, with the distance A F, cross F D, at E; through the points C and E draw C E G, cutting A H at G; make F D equal to C G, then will A F and F D be the two semiaxes; then proceed as before. Fig. 5 and 6, are described in the same manner.

## PLATE 68. FIG. 1 and 2.

*The semitransverse and semiconjugate axes being given, to describe the moulding.*

Proceed as in Page 24. Prob. 6, Geometry, and you will have the contour of the moulding required.

## FIG. 3.

*To describe the semirecta, the perpendicular height H L being given, and its projection L I.*

Complete the rectangle I L H F, and divide the whole rectangle into four equal rectangles; then inscribe the concave quadrant of an ellipsis, in the rectangle I K C B, and a convex quadrant in the rectangle C G H D, and it is done.

## FIG. 4.

*To describe a cimareversa, the point A being nearly the greatest projection at the top; D, the extremity of the curve at the bottom; and D C a line parallel to a tangent, at the point of junction of the opposite curves.*

Draw A C at right angles to C D, cutting C D at C, and complete the rectangle, A C D E; then proceed as before, but in a contrary direction, and you will have the contour required.

## FIG. 5.

*To describe an echinus, having the depth of the moulding C D, the greatest projection at D, and to be quirked at the top and bottom.*

Complete the rectangle C B A, and proceed as in Problems XVIII, XIX, and XX, Page 31 and 32, Geometry.

## FIG. 6.

*To describe a scotia.*

Join the ends of each fillet by the right line A B; bisect A B at D; through D draw C D E, parallel to the fillets, and make C D and D E equal to the depth of the scotia; then will A B be a diameter of an ellipsis, and C E its Conjugate. Proceed as in Prob. IV, Page 24, Geometry.

# MOULDINGS

Pl. 68.

Fig. 1.

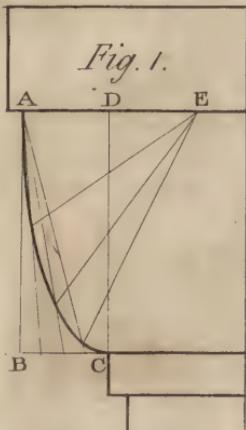


Fig. 2.

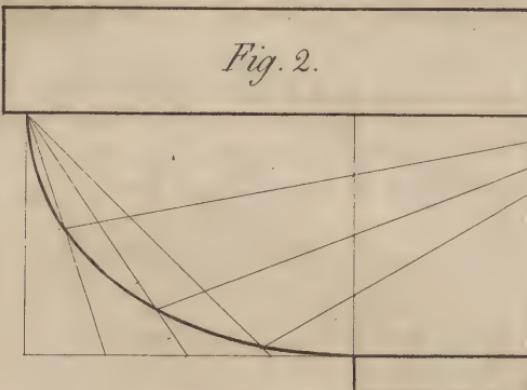


Fig. 3.

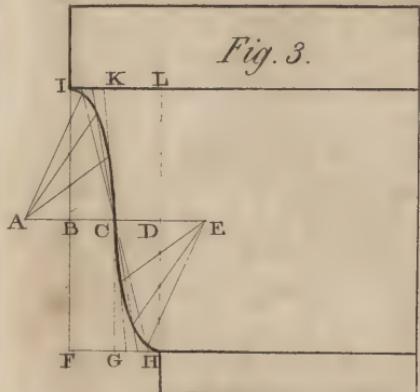


Fig. 4.

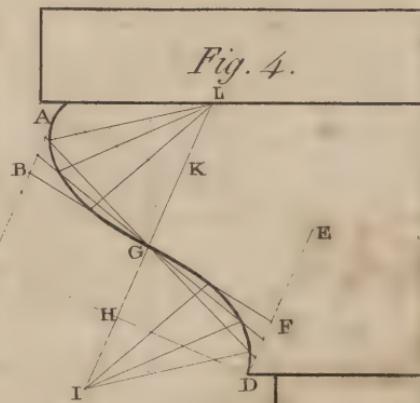


Fig. 5.

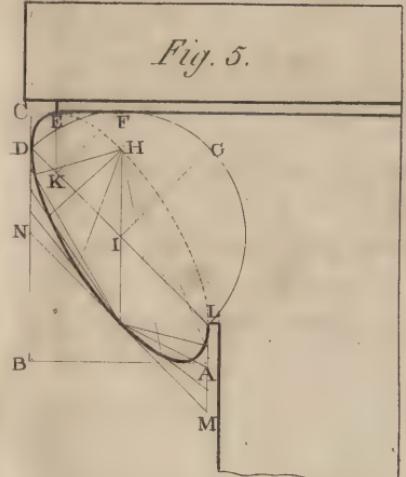
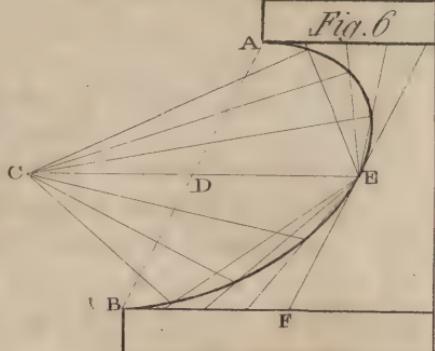
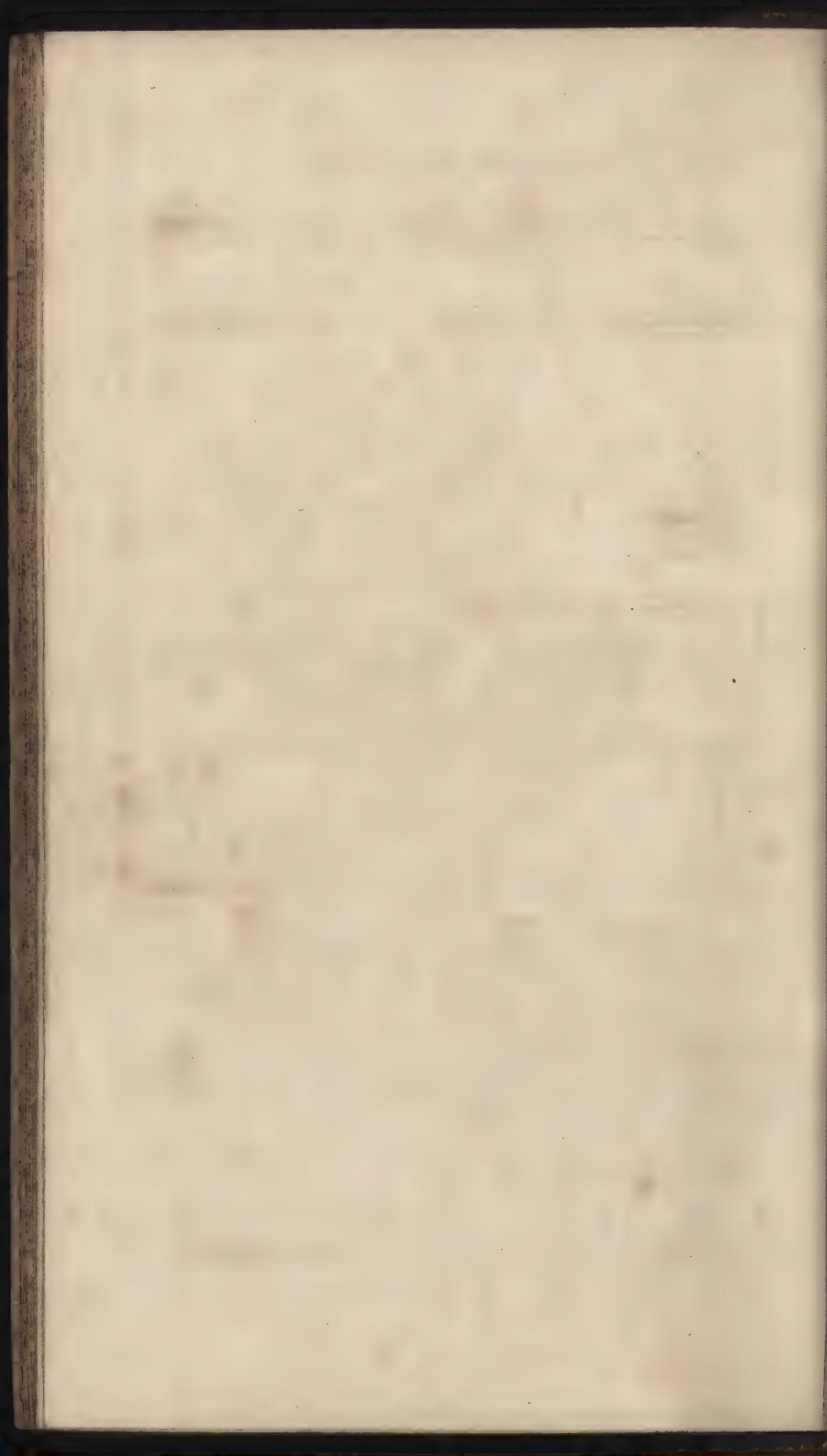


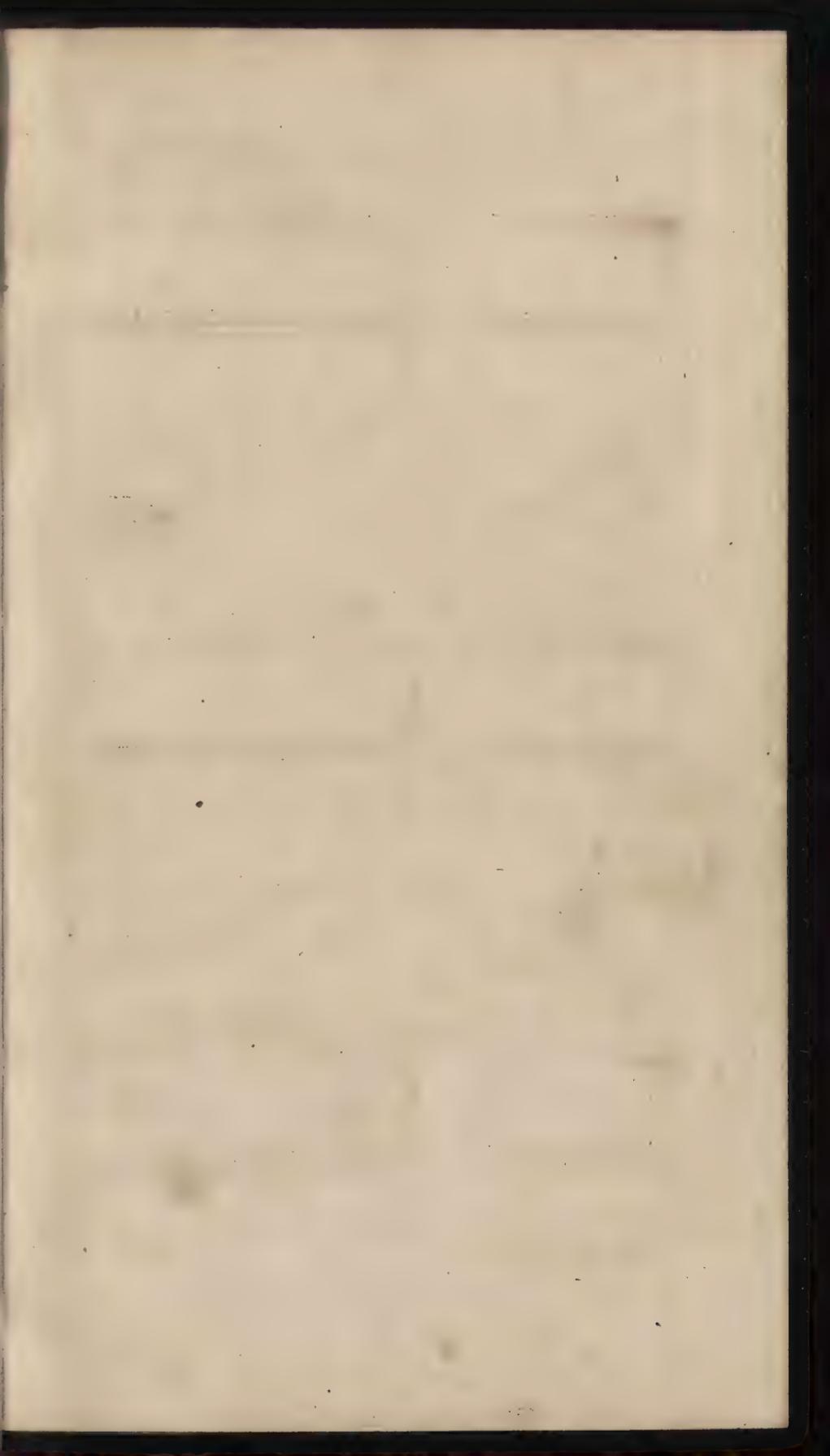
Fig. 6.



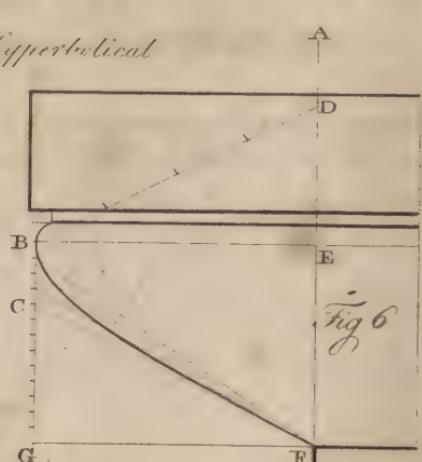
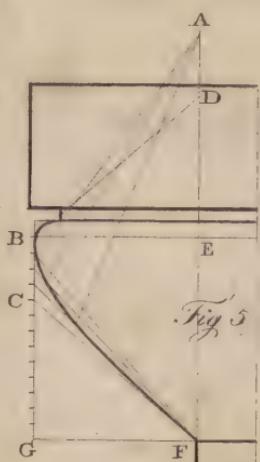
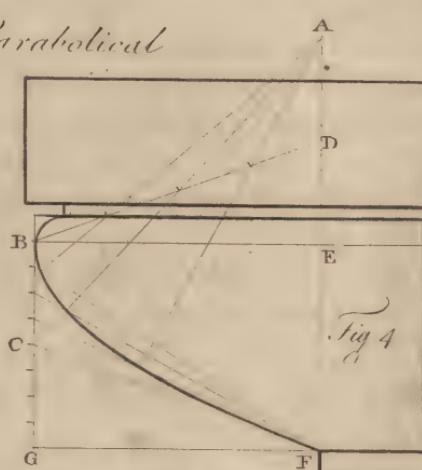
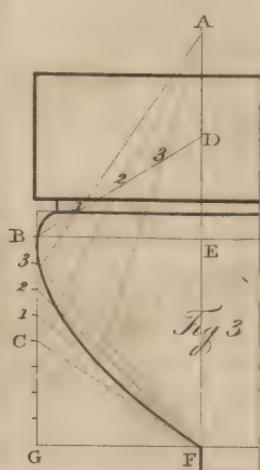
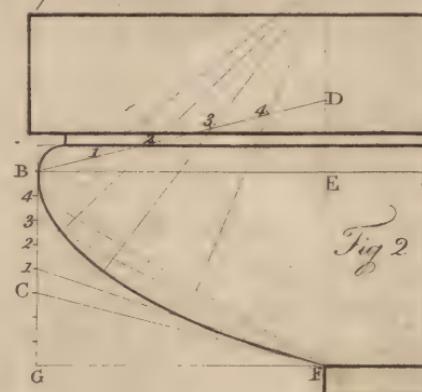
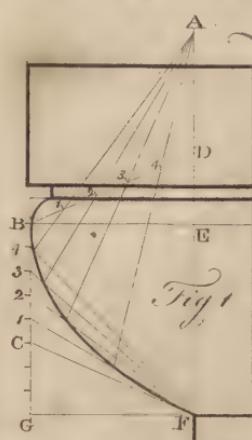
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Published Oct. 1. 1795. by P. Nicholson & Co.





## MOULDINGS FROM CONIC SECTIONS



## PLATE 69. FIG. 1, 2, 3, 4, 5 and 6.

*How to describe Grecian mouldings, whether elliptical, parabolical, or hyperbolical ; the greatest projection at B being given, and the tangent C F at F the bottom of the moulding.*

Draw G F, a continuation of the upper side of the under fillet; through B draw B G, perpendicular to B F, cutting it at G, and the tangent F C at the point C; also through B draw B E, parallel to G F; and through F draw F E D A, parallel to G B, cutting B E at E; make E A equal to E F, E D equal to C G, and join B D; then divide each of the lines B D and B C into a like number of equal parts; from the point A, and through the points 1, 2, 3, 4 in B D, draw lines; also from F, through the points 1, 2, 3, 4 in B C, draw lines cutting the former, which will give points in the curve.

If the point C, where the tangent cuts the line B G, be less than one half of B G, from G, the moulding will be elliptical as in Fig. 1 and 2.

If G C be one half of B G, the moulding is parabolical, as in Fig. 3 and 4.

If G C be greater than half of B G, then the moulding is hyperbolical, as in Fig. 5 and 6.

By this means you may make a moulding to any form you please, whether flat, or round.

## PLATE 70. FIG. 1 and 2.

*How to draw a combination of mouldings, for the base of a column, to be placed above or below the eye; so that the contours of the mouldings shall appear nearly similar to the mouldings of a base, situate on a plane, passing through the eye, parallel to the horizon.*

Let A B, C D, E F, H G, I Q, be lines tending to the eye; describe each moulding to touch these lines, as is taught in the foregoing problems; that is, describe the hollow as in Fig. 4, Plate 66, Page 12, to touch the lines B A, and A S, at the given point B; then describe the semiellipsis within the rectangle E F H G, for the scotia; Page 14, Plate 68, Fig. 6.

The under torus will be described as follows:

Bisect the diameter I L at T, and continue the upper side of the plinth from L to U; bisect Q U at P, and through the points P and T, draw P T O: join I P, and bisect it at R; through the points Q and R, draw Q R N, cutting P O at N; make N O equal to N P; then proceed as in Prob. 6, Page 24, Geometry, and it will be the torus.

If the mouldings in a building were executed according to the principles of optics by the above rules, the effect of every moulding would be seen, whether above or below the eye, when it is situate at a judicious distance from the object, much more perfectly than in common cases, where some of the mouldings are entirely hid by the projection of others standing before them.

## MOULDINGS

Fig. 1.

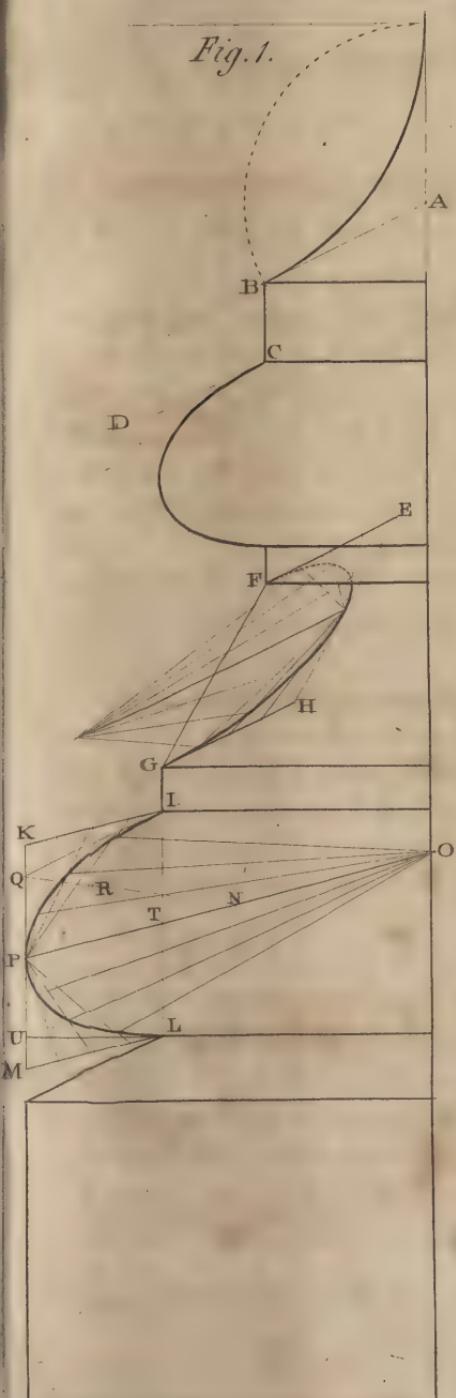
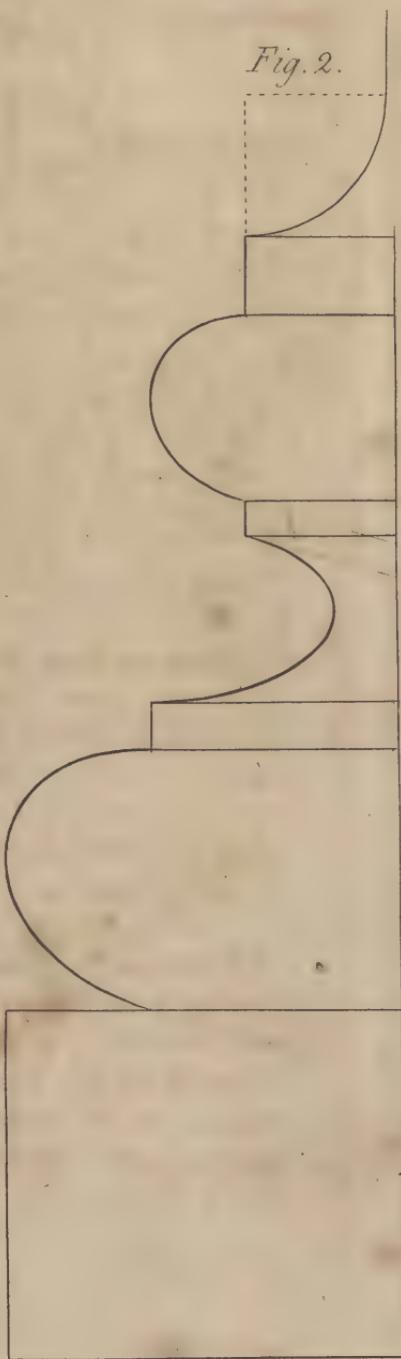
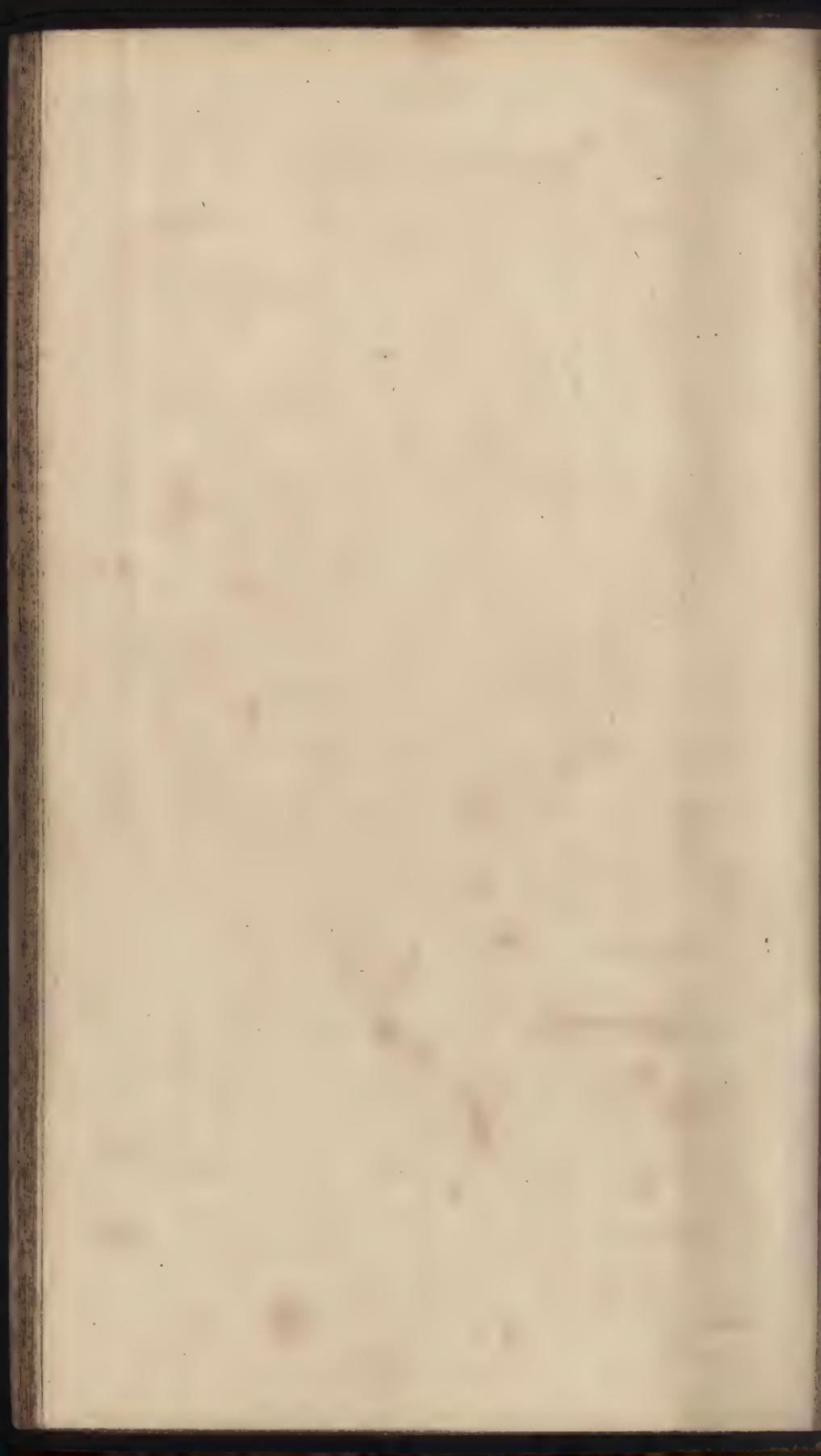
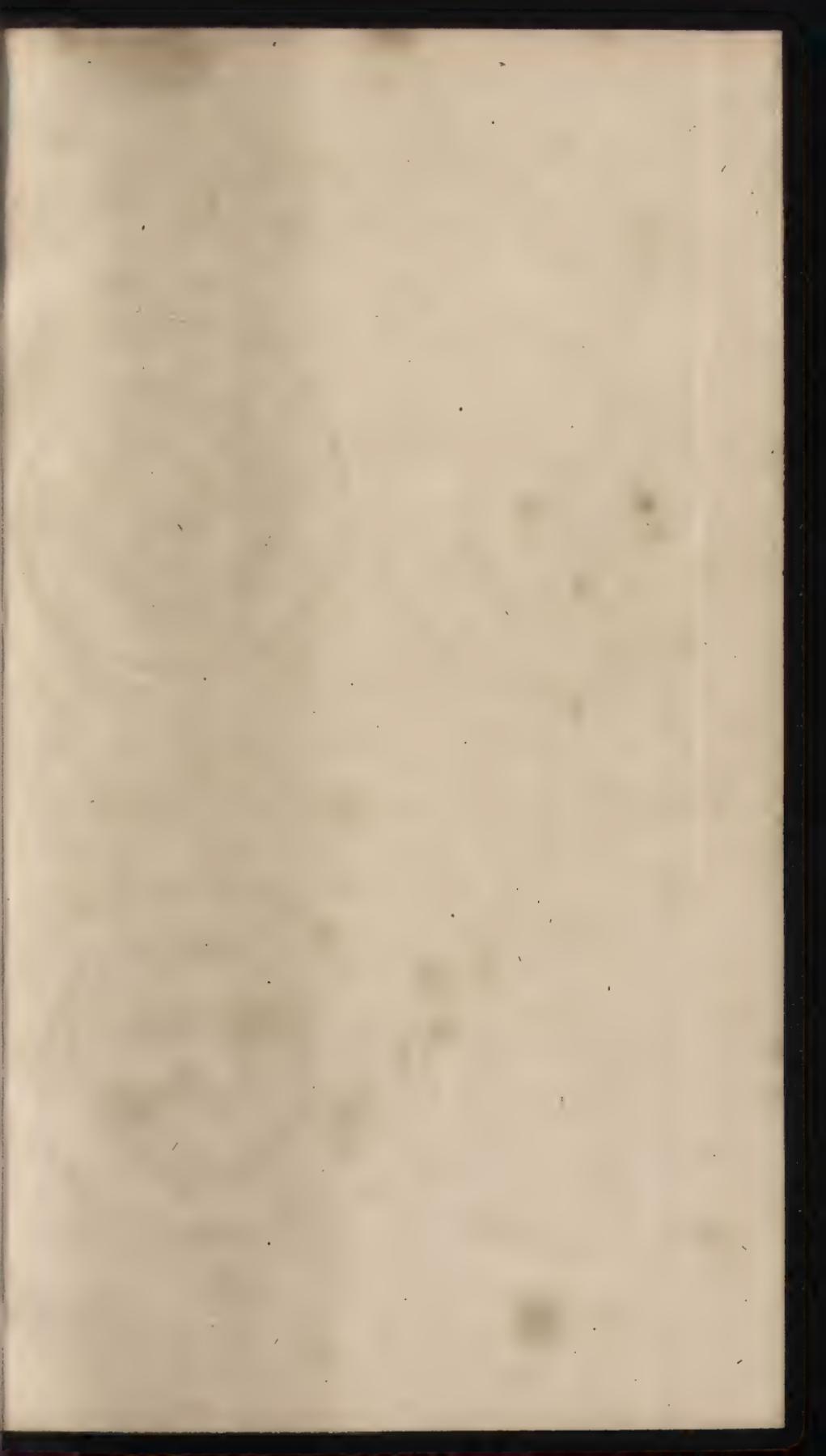


Fig. 2.





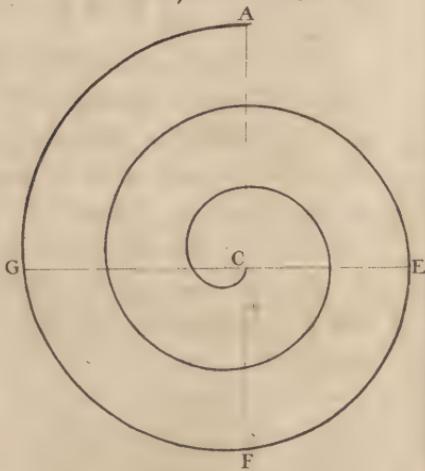


SPIRAL LINES.

Def. 1. Fig. 1.



Def. 5. Fig. 2.



Def. 6. Fig. 3.

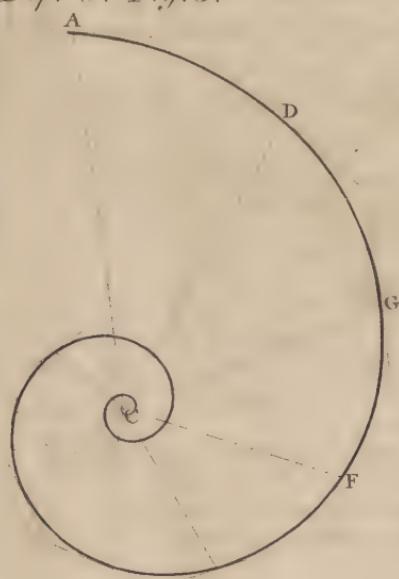
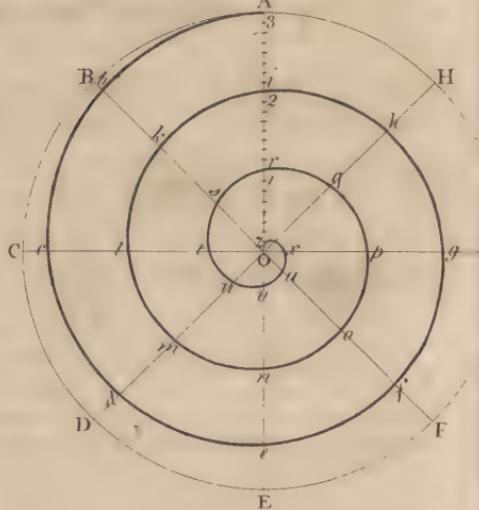


Fig. 4.



Drawn by P. Nicholson

Engraved by W. Scoury

## SPIRAL LINES.

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### DEFINITIONS.

#### PLATE 71. FIG. 1.

1. If round a fixed point another be supposed to move continually, approaching or receding from the fixed point, according to some law, the figure so described is called a Spiral.\*
2. If the moving point has gone once round the fixed point, the spiral is said to have one revolution; and if twice round, it is said to consist of two revolutions; and so on.
3. The fixed point is called the centre of the spiral.
4. Any straight line drawn from the centre of the spiral, and terminated by the curve, is called an ordinate.

#### FIG. 2.

5. If the radius C A be moved uniformly round the centre C, and at the same time a point be supposed to move uniformly along the radius from C to A, so that both motions begin and end together, the curve E F G A is called the spiral of Archimedes.

#### FIG. 3.

6. If the curve F G D A be such, that if it was every where cut by the radius C F, C D, C G, C A, &c. all the angles made by tangents at the points F, D, G, A, with the radii C F, C D, C G, C A, &c. may be equal, then the curve is called the Logarithmic, or Proportional Spiral.

\* Although a spiral, strictly speaking, signifies a line drawn round the surface of a cone, which line is continually approaching nearer to the axis as it comes nearer to the vertex of the cone, yet Mathematicians define it as above on a plane.

## PROBLEM I.

To find any number of points in the Spiral of Archimedes, for tracing the curve to any number of revolutions, having the centre given, and the greatest distance that the spiral is to recede from the centre.

On the centre of the spiral, with a radius equal to the greatest distance, describe a circle; divide the circumference, beginning at the greatest distance, into any number of equal parts; from each of these points in the circumference, draw lines to the centre; divide any one of the radii into as many parts as there are to be revolutions; and divide each of these parts again into as many equal parts as the circumference of the circle is divided, and call this the scale; then from the centre set off the radius, made less by one of these parts, upon the next radius to that which passes through the greatest distance; next take the radius made less by two, and apply that from the centre on the next radius; again take the whole scale, made less by the three parts, which set from the centre on the next radius. Proceed in this manner, making each succeeding radius less, by one part of the scale, than the preceeding one, until you arrive at the centre; then a curve being drawn through all the points in the radii will form the Spiral required.

## EXAMPLE.

FIG. 4.

*To draw the spiral of Archimedes to the three revolutions; the centre O being given, and the point A, which is the greatest distance from the centre.*

Join A O on O, with the radius O A, describe a circle; divide the circumference, beginning at A, into any number of equal parts, suppose into eight, at the points A, B, C, D, E, F, G, H; through all these points, draw lines toward the centre C; divide the radius C A into three equal parts, because the curve is to have three revolutions; then divide each of these parts into eight equal parts, because the circle A B C D E F G H, is divided into eight equal parts, by the radii O A, O B, O C, O D, O E, &c. then the radius A O will be divided into twenty-four equal parts; make O b, in the next radius, equal to twenty-three parts; O c, equal to twenty-two, O d, equal to twenty one, O e, equal to twenty, and proceed in this manner till you arrive at the centre C; the curve being drawn through the points A, b, c, d, e, f, g, &c. will give the spiral required.

*Note,* Although in this example the curve is only divided into eight equal parts, which may be quite sufficient for a small spiral; yet where great accuracy is required, the circumference may be divided into sixteen or more parts.

## PROBLEM II.

## PLATE 72. FIG. 1.

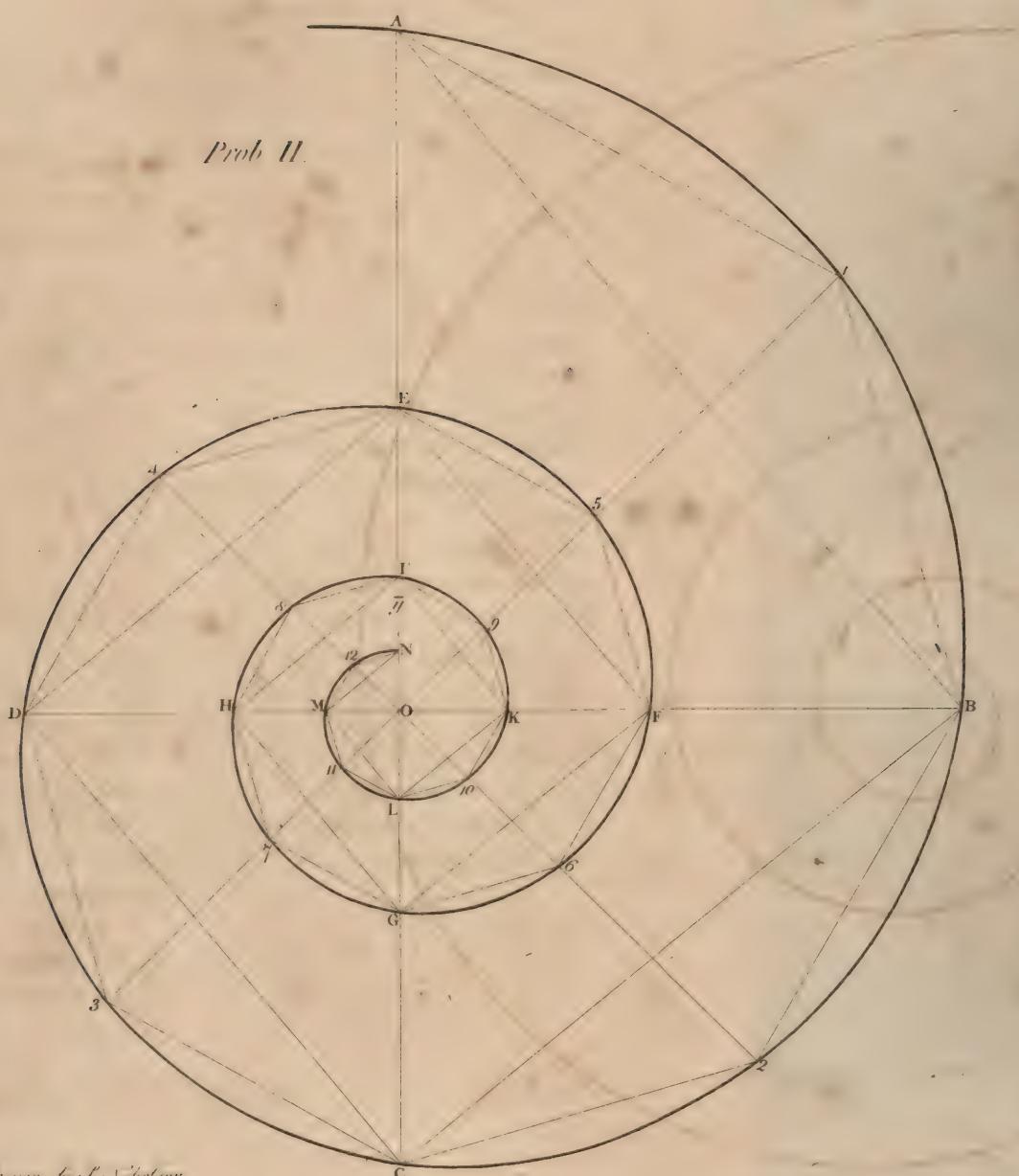
The height  $A\ C$ , passing through the centre  $O$ , being given, and the centre  $O$ , to find any number of points in the proportional spiral.

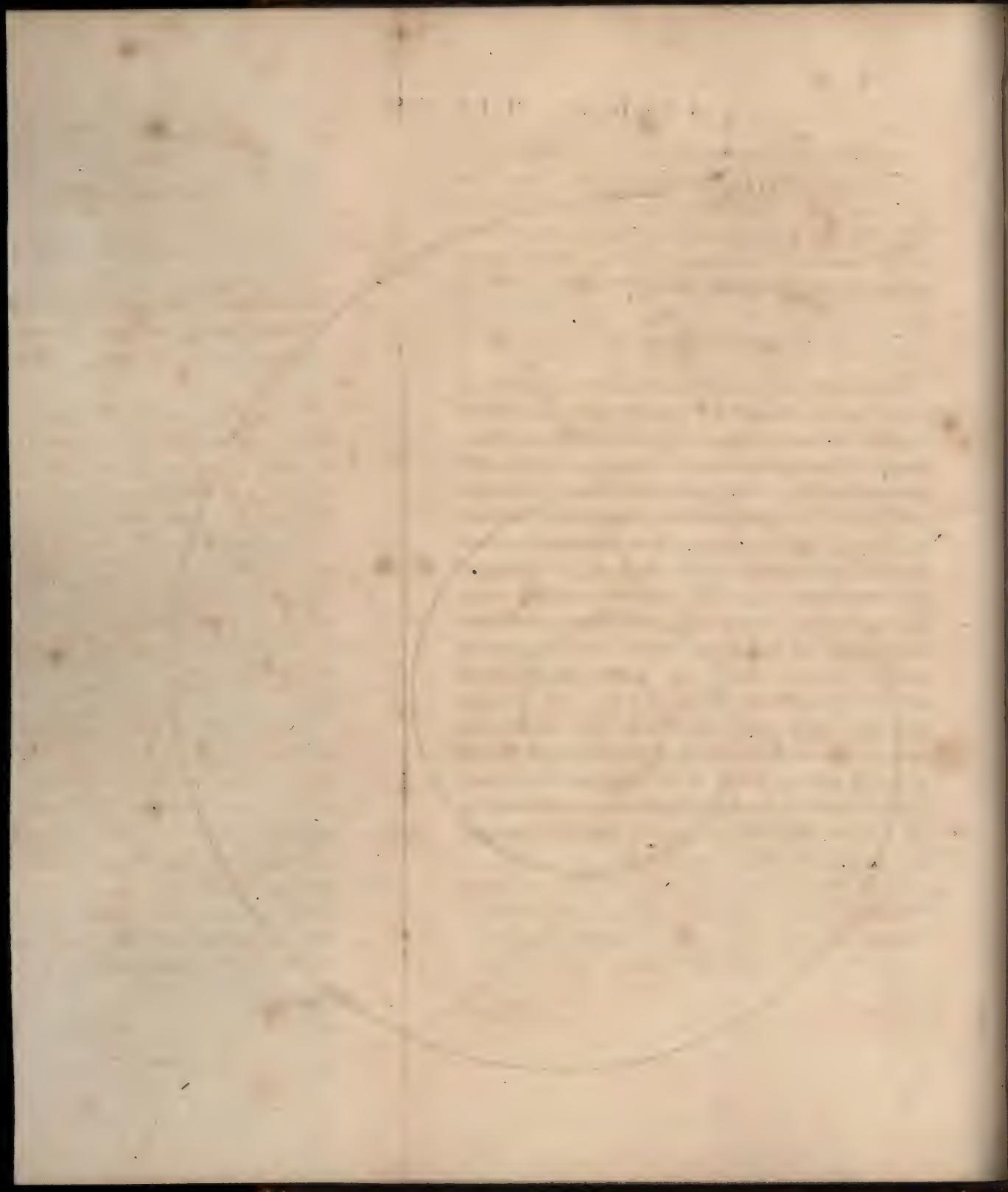
Through  $O$ , draw  $B\ D$  perpendicular to  $A\ C$ ; find  $O\ B$  a mean proportional between  $O\ C$  and  $O\ A$ ; that is, bisect  $A\ C$  at  $y$ ; on  $y$  as a centre, and with the distance of  $y\ A$ , describe an arc, cutting  $B\ D$  at  $B$ ; join  $A\ B$  and  $B\ C$ ; then through  $C$ , draw  $C\ D$  parallel to  $A\ B$ , cutting  $B\ D$  at  $D$ ; through  $D$ , draw  $D\ E$  parallel to  $B\ C$ , cutting  $A\ C$  at  $E$ ; through  $E$ , draw  $E\ F$  parallel to  $A\ B$ , cutting  $B\ D$  at  $F$ . Proceed in this manner to the end of the last revolution, and a point will be obtained at the beginning and end of every quarter of a revolution.

Then to find any number of intermediate points, bisect the angles  $A\ O\ B$  and  $B\ O\ C$  by the lines  $1\ 3$  and  $2\ 4$ , and continue those lines to the other side of the centre  $O$ , and the angles  $C\ O\ D$  and  $D\ O\ E$  will also be bisected. Make  $O\ 1$  a mean proportion between  $O\ A$  and  $O\ B$ ; and  $O\ 2$  a mean proportion between  $O\ B$  and  $O\ C$ ; join  $A\ 1$ ,  $1\ B$ ,  $B\ 2$ ,  $2\ C$ ; then draw  $C\ 3$  parallel to  $A\ 1$ ,  $3\ D$  parallel to  $B\ 1$ ,  $D\ 4$  parallel to  $B\ 2$ ,  $E$  parallel to  $C\ 2$ ; that is, each parallel to that subtending its opposite angle.

Again

## SPIRAL LINES





Again draw E 5 parallel to A 1, F 5 parallel to B 1; F 6 parallel to B 2, G 6 parallel to C 2; proceed in this manner by continually drawing lines parallel to A 1, 1 B, B 2, and 2 C, till a point is found in the middle of each quarter; a curve being drawn through these points will give the spiral required.

### SCHOLIUM.

Although by the preceding methods any number of points may be found, the first of these is not well adapted for a spiral when applied to architectural purposes, because on account of the space comprehended between the line of the spiral being every where parallel, it is thought ungraceful by Architects. Method 2, is the most perfect that can possibly be put in practice; as the revolutions every where divide the radius in a continued proportion, and consequently every space will have the same ratio as its distance from the centre; by this means you may describe as great a number of intermediate curves as you please, and the distance of those lines will have every where the same proportion taken upon a line drawn to the centre; but as many Architects may think this method troublesome to put in practice, I will, in the following, show more general methods for describing curves of the kind, to any number of revolutions with a compass, than has been hitherto shown.

## PROBLEM III.

## PLATE 73. FIG. 1, 2, 3, 4.

To find centres for drawing spirals to any number of revolutions; the centre C being given, and also the perpendicular line C G.

Make C D both perpendicular and equal to C G; divide C D, beginning at C, into as many equal parts as you intend to have revolutions; divide the first of these from the centre C, that is C 1, into two equal parts, at the point I; through the points D and G, draw D B and G B respectively parallel to C G and C D, cutting each other at B; in the line B G make G A equal to G B; join C A and C B; through I, draw I E parallel to C A, cutting B D, produced at E; through the same point I, draw I F parallel to C B; then through the point D, fig. 1; and 1, D, fig. 2; 1, 2, D, fig. 3; 1, 2, 3, D, fig. 4, &c. draw the lines parallel to B E cutting the diagonal E I and B C. From the points where the diagonals are cut, draw lines parallel to B A, cutting the other diagonals A C and F I; and through the points which are cut in the diagonals A C or F I, draw lines again parallel to B E, and it will complete the centres for turning the spiral.

*Note.* Fig. 1, is for one revolution; fig. 2, for two revolutions; fig. 3, for three revolutions; fig. 4, for four revolutions; fig. 5, for five revolutions; and fig. 6, for six revolutions.

Fig. 1.

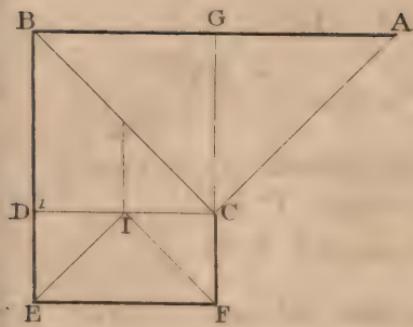


Fig. 2.

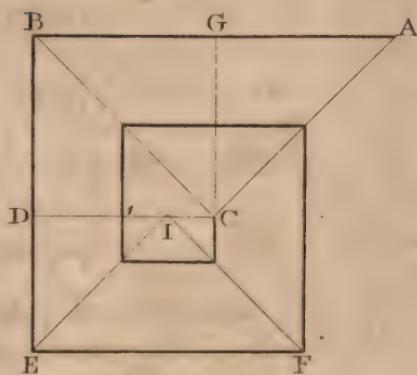


Fig. 3.

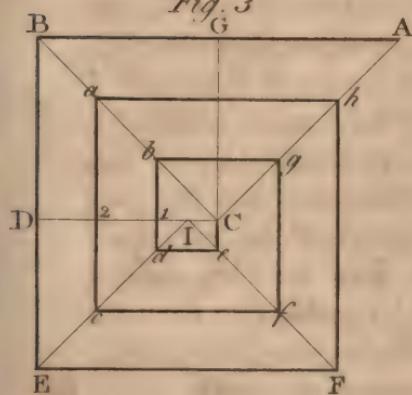


Fig. 4.

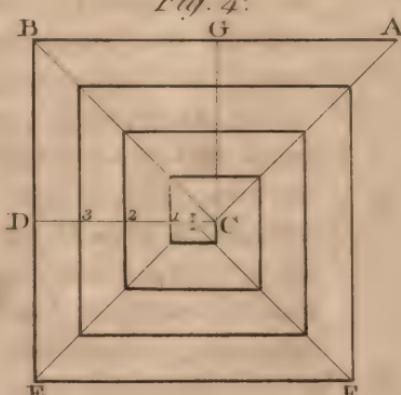


Fig. 5.

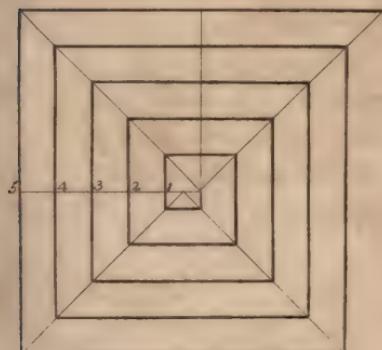
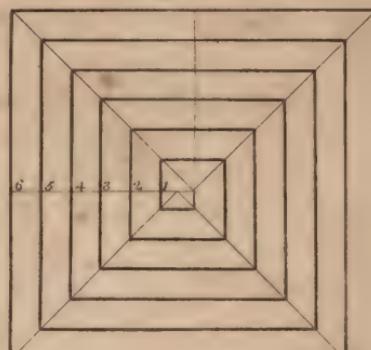
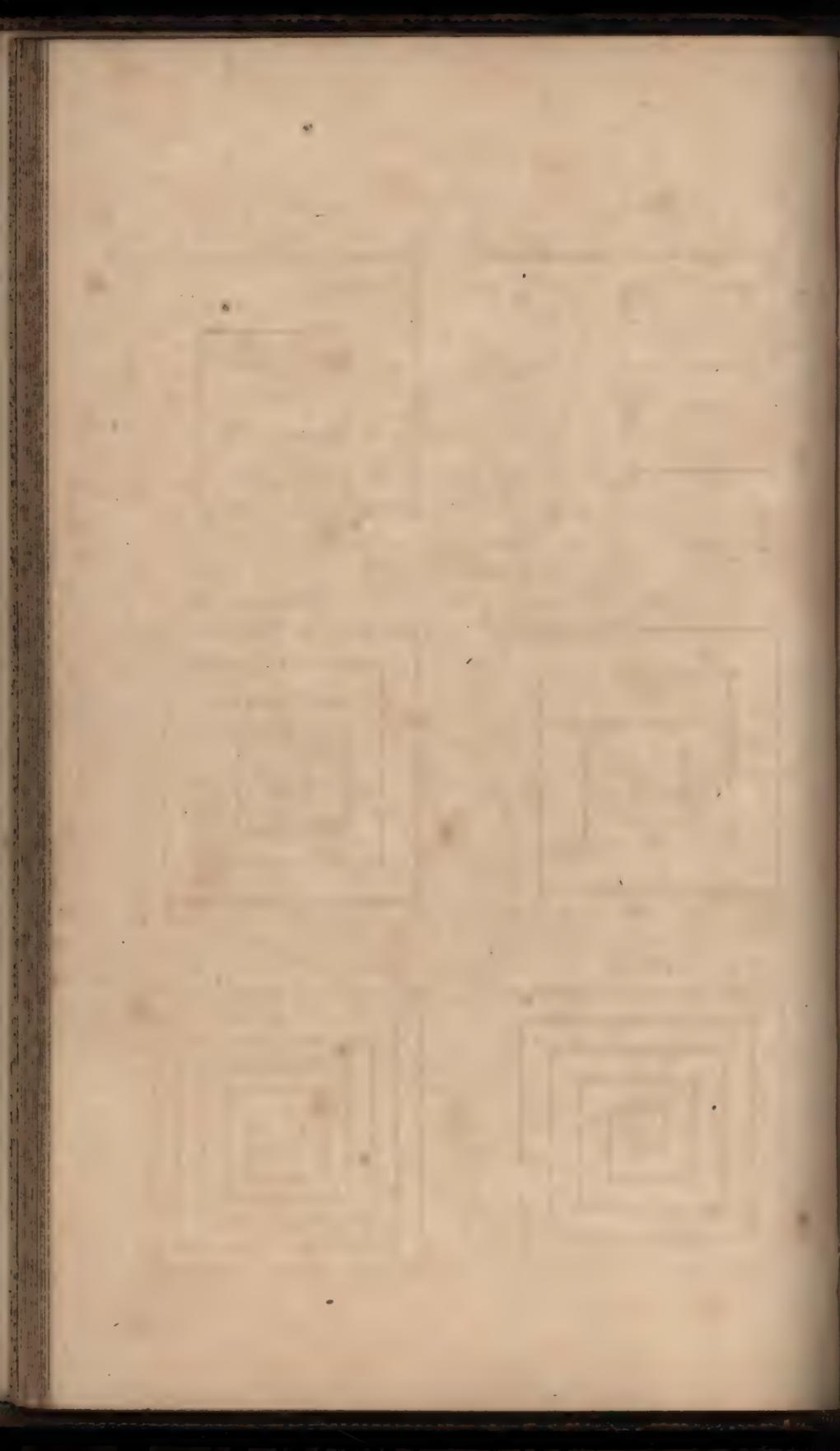
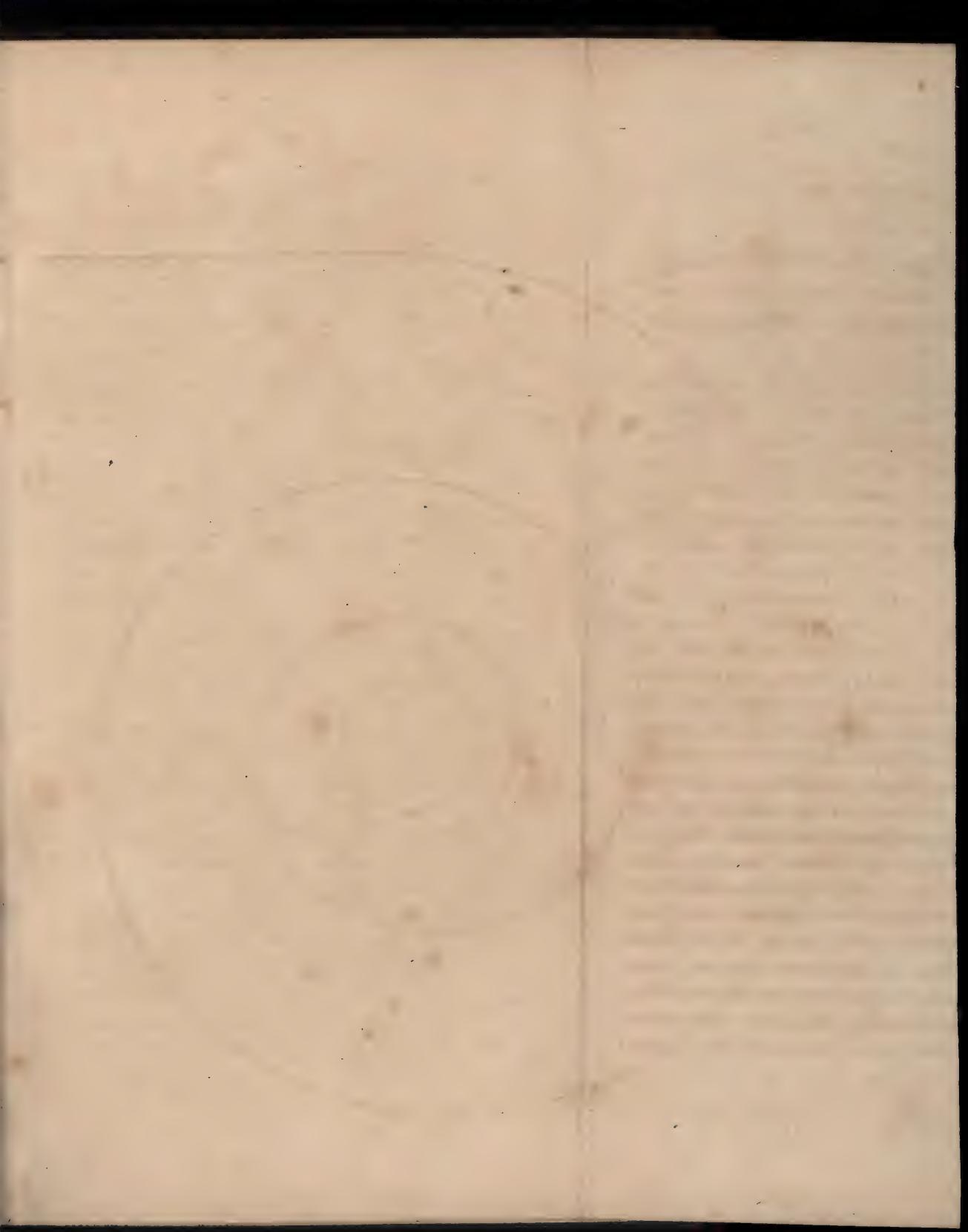


Fig. 6.

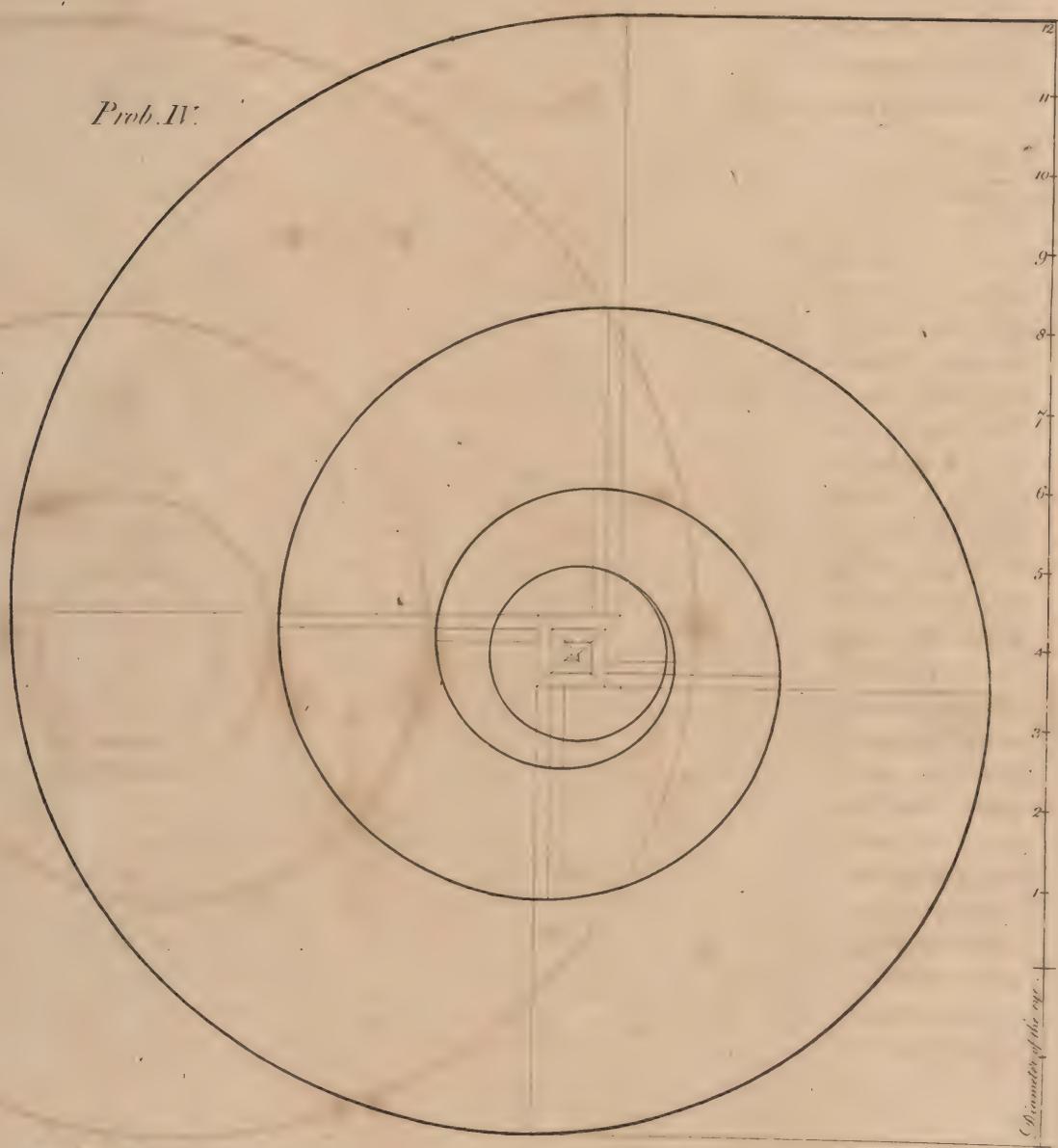






Pl 74.

SPIRAL LINES



Invented by P. Nicholson.

London Published Sept 1796 by P. Nicholson & C°

## PROBLEM IV.

## PLATE 74.

To draw a spiral line to touch a given circle, whose centre is the centre of the spiral, to any number of revolutions; the whole height being given.

From either end of the whole height of the spiral cut off the diameter of the given circle; divide the remainder into as many equal parts as there are to be revolutions in the spiral, and divide each of those parts again into four others; so that the remainder, or difference between the given circle and the height, will be divided into four times the number of revolutions; then take half the number of these parts and one part more, together with half the diameter of the eye, and set it from the top of the perpendicular downwards, it will give the centre of the volute; or take half the number of parts made less by one part, and set it from the bottom upwards, will also give the centre; take half of one of the parts, and set it from the centre, cutting the perpendicular or height of the volute upwards; through that point, draw a horizontal line; take half one of the parts, and set it on each side of the perpendicular, on the horizontal line; from these two points draw diagonals to the centre; through the centre, draw another line parallel to the horizontal line; through the upper end of each diagonal, draw lines parallel to the perpendicular, cutting the horizontal line that passes through the centre, into two equal parts; divide each of those parts into as many equal parts as you intend to have revolutions. If the volute is intended to be on the left hand, divide the part next

to

to the centre on the same side, into two equal parts but for the right on the contrary ; from the point of the bisection, draw two lines parallel to the diagonals downwards ; then through each of the divisions on the line which passes through the centre, draw lines parallel to the perpendicular, cutting the diagonals at both ends of these perpendicular lines ; then join the opposite points of each diagonal by horizontal lines, and the centres will be completed upon each angle of the fret ; begin at the right hand on the upper centre, extend the compass to the height of the perpendicular, and describe the quadrant of a circle to the left hand ; then set the compass on the next centre on the left hand, and extend the other leg of the compass to the end of the quadrant where you left off in the last quadrant : go the same way round to the next centre, and proceed in this manner till you arrive at the last quadrant, which ought to touch the given circle on the upper side upon the perpendicular. Lastly, on the centre of the spiral, and the other foot extended to the distance that the last quadrant cuts the perpendicular, describe a circle, and the spiral will be completed.

#### SCHOLIUM.

1. The common method for describing a spiral is imperfect at the meeting of every fourth quadrant with the next succeeding one ; or, which is the same thing, at the meeting of each two quadrants at the beginning of every revolution, excepting the first, will not touch a straight line at the points of junction, and consequently will form a small angle at their meeting ; and for this reason the whole curve of the spiral will appear lame.

2. In

2. In Goldman's volute, the meeting of every two quadrants will every where touch a straight line at the points of their junction ; but the diminution of the space is no regular proportion ; as one half diminishes very rapidly, and the other half being almost equidistant ; which fault is more discernible in this, than the lame curve of the common method, which is not perceptible to the eye but only to sense.

3. In the method which I have shown for describing spirals with a compass, the meeting of every two quadrants will every where touch a straight line at the point of their junction, and the spaces are all in arithmetical proportion at the beginning of each quadrant ; consequently this method is more perfect than the two former, as the defects of both are remedied in this.

The methods for describing volutes, made use of by De l'Orme and Goldman, are very limited, and not so general as may be wished for, to answer the different purposes that they may be applied to in Architecture. I shall therefore show how to describe their volutes in a more general manner than has hitherto been done ; so that the eye may be of any size required, and the volute contain any number of revolutions in a given height.

## PROBLEM V.

How to describe a spiral in the manner of De l'Orme's volute ; the perpendicular height being given.

Divide the whole height into eight equal parts ; on the fourth from the bottom, or fifth from the top, as a diameter, describe a circle ; through its centre draw a line at right angles to the cathetus, which will cut the eye in four points : join the extremities of these points, and a square will be inscribed in the eye ; bisect any two adjoining sides of the square, and draw lines from these points through the centre of the eye, to cut the opposite side of the square ; divide each of these lines, which are terminated by the sides of the square, into six equal parts : then the four outside points, bisecting each side of the square, are centres for the first revolution, the next four are centres for the second revolution, and the next four, viz. those next to the centres, are centres for the third and last revolution : then each of these four centres will form four other squares, having their side parallel to the perpendicular and horizontal line ; then you may proceed to describe each quadrant of the volute as follows :

If the volute is to be on the right hand, begin with the left-hand centre on the top of the outside square, extend the other foot of the compass to the top of the volute, and describe an arc towards the right hand ; then move your compass to the next angle of the same square on the right hand, and extend the other to the end of the last quadrant, and turn an arc downwards, cutting a per-

a perpendicular from its centre, the foot of the compass remaining in that point; contract their opening equal to the side of the square on the same straight line, till you come to the next centre; describe another quadrant, meeting the side of the square produced; again contract the compass the side of the square as before, and describe another quadrant, until it cuts the perpendicular side of the square produced from that centre; then there is one revolution described; now proceed with the second and third square in the same manner, until you describe the last quadrant, which will cut the eye on the upper side.

---

### PROBLEM VI.

How to describe the spiral to any given height, and to terminate at the end of any given number of revolutions in the circumference of a circle, whose diameter is given.

Divide the height, made less by the diameter of the given circle, in two more parts than four times the number of revolutions that the spiral is intended to have; that is, if the spiral is to have one revolution, it must be divided into four times one, and two more, that is, into six equal parts; and if two revolutions, it will be divided into four times two, and two, which is ten equal parts; or if three revolutions, it will be divided  $4 \times 3$ , and two, which is fourteen equal parts; then take half the number of these parts, and one more, together with half the diameter of the eye, and set it from the top of the spiral downwards to give the centre

of the spiral ; or take half the number of those parts, made less by one, together with half the diameter of the eye, and set it on the perpendicular line from the bottom upwards, which will give the centre of the spiral as before ; then construct a square, whose centre is the centre of the spiral, having two of its sides parallel to the perpendicular, and consequently the other two at right angles to it ; each side of the square, being equal to one of the equal parts before mentioned ; draw the diagonal of the square, which will cut each other in the centre of the spiral ; now divide each diagonal into twice as many parts as there are to be revolutions, which will give the centres of the spiral ; and then proceed as in the last problem.

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### PROBLEM VII.

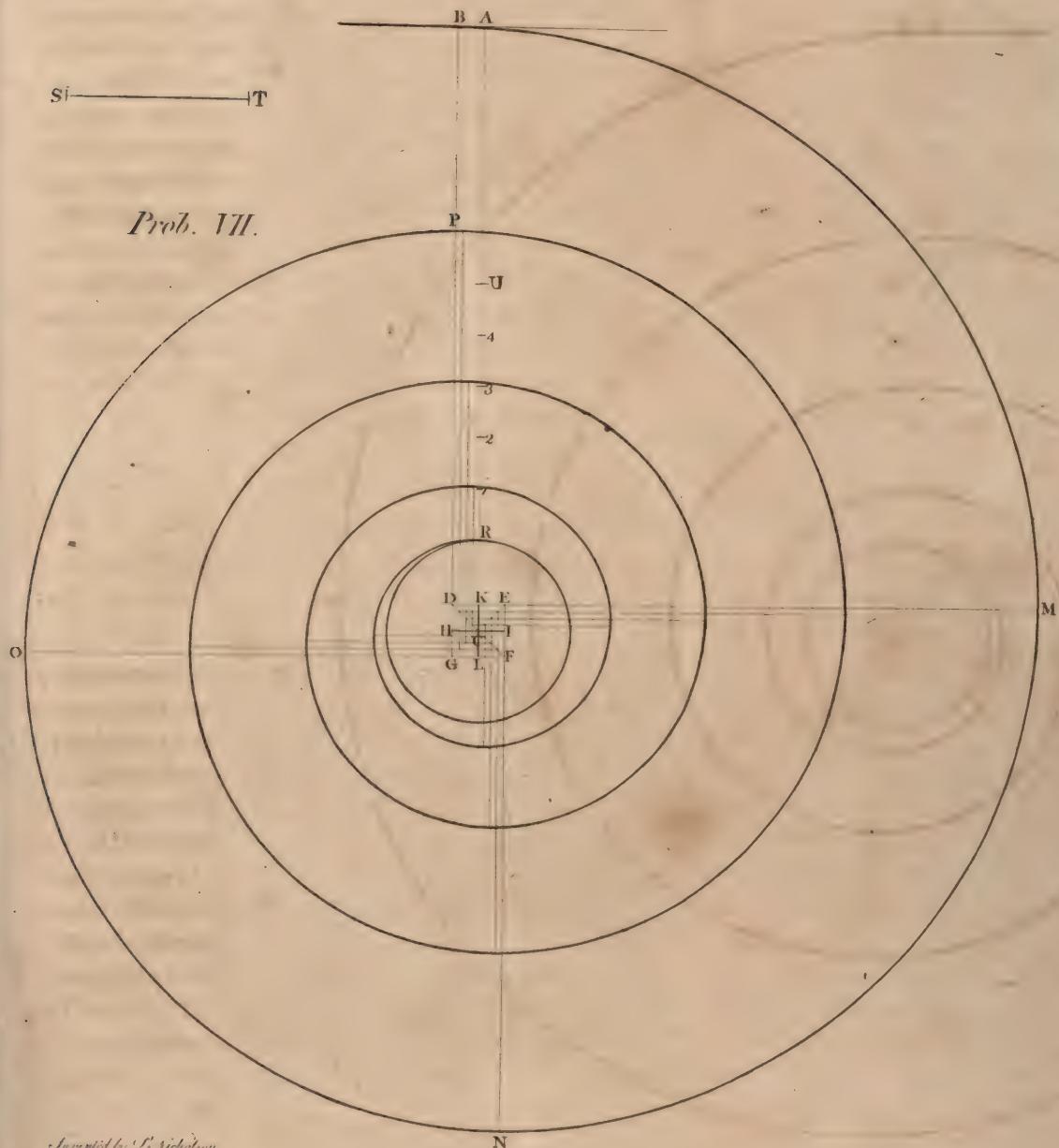
The centre C of the spiral being given, the perpendicular height C A above the centre, and diameter B D of the eye ; to describe the spiral.

On the centre C, with one half of B D, describe a circle, cutting A C at R ; through C draw H I perpendicular to A C ; divide A R into two equal parts at I ; divide I R into one more part than the number of revolutions, then set half of one of these parts from the centre upon each of the lines A C and H I : that is, from C to K, from C to I, from C to L, and from C to H : and through these points complete the square D E F G, whose sides D E, F G, are parallel to H I ; and D G, E F, are parallel to A C ; draw the diagonals D F and G E, then divide C D, C E, C F, and C G, each into as many equal parts as there are to be revolutions,

# SPIRAL LINES

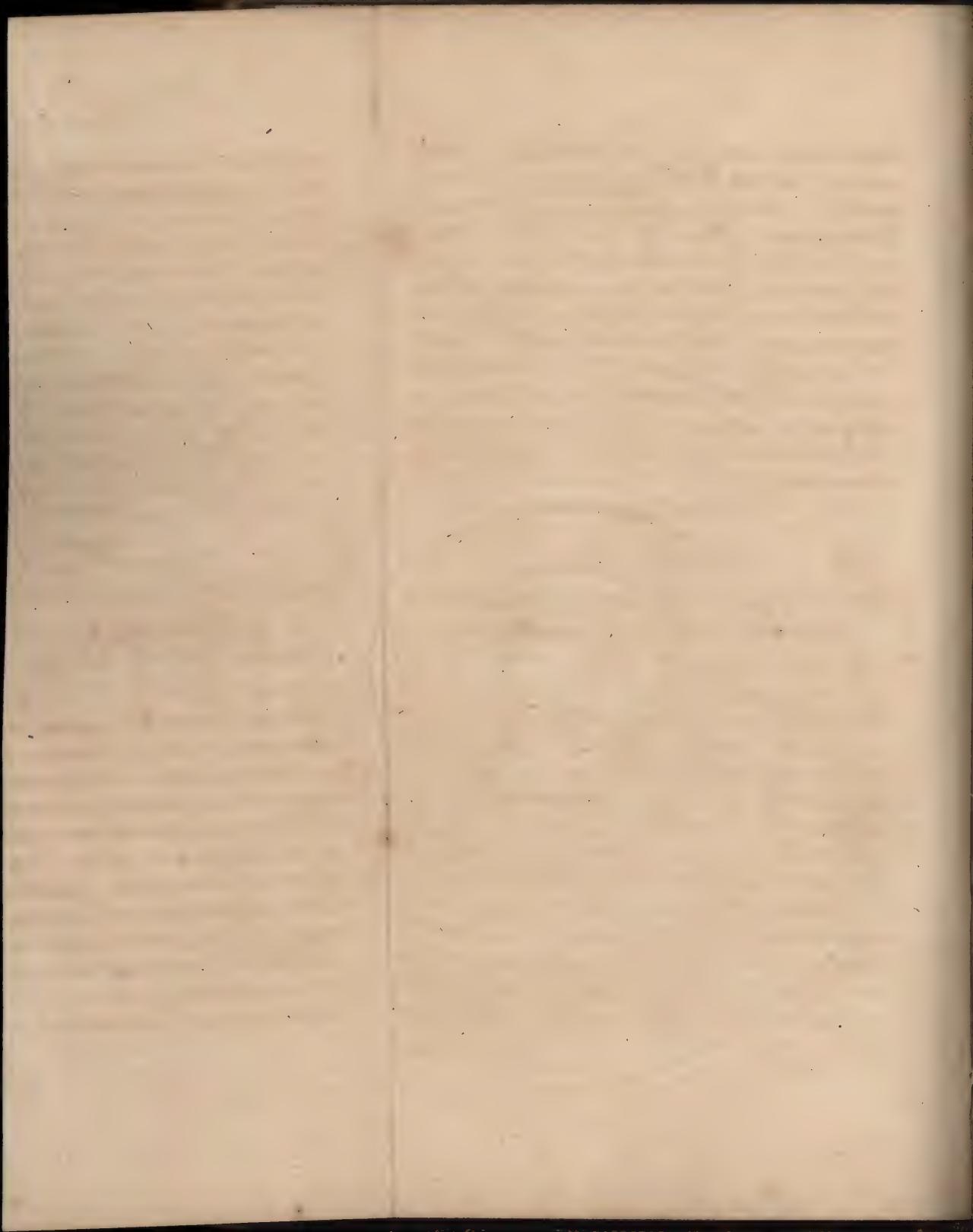
Pl. 75.

Prob. III.



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lutions, and through these points draw squares, whose sides are parallel to the sides of the square D E F G ; through A, draw A B parallel to H I, produce the side of the square D G, cutting A B in B ; then on D as a centre with the distance D B, describe the quadrant B M, cutting D E at M ; on E, with E M, describe the quadrant M N, cutting E F produced at N ; on F, with the distance F N, describe the quadrant N O, cutting F G at O ; on G with the distance G O, describe the quadrant O P, which will make one revolution, and proceed in the same manner with all the other revolutions, the centres always falling on the angles of the next square.

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### PROBLEM VIII.

How to describe a spiral line in the manner of Goldman's volute, to any given height.

Divide the height I W of the volute into eight equal parts, I being at the top ; then on the fourth part from the bottom call A B, or the fifth from the top, or B A as a diameter ; describe a circle, whose centre call C ; divide B A into four equal parts, and from the centre, set one of these parts on the perpendicular from C to H downwards, and from C to E upwards, and construct a square E F G H, whose side is F G on the right or left side of H E, according as it is intended to be a right or left-handed volute ; join C G and C F, divide C H, C E, C G, and C F, each into three equal parts respectively at *q, m ; n, i ; p, l ; o, k* ; then will E, F, G, H, *i, k, l, m, n, o, p, q*, be the centres for describing each quadrant. On E, with the distance E I, describe the quadrant.

quadrant I K, cutting E F produced at K; on F as a centre with the distance F K, describe the quadrant K L, meeting F G produced at L; on G, with G L, describe the quadrant L M, cutting G H at M; on H, with the distance H M, describe the quadrant M N, cutting the cathetus at N. In this manner proceed with the centres  $i, k, l, m$ , for the second revolution; and  $n, o, p, q$ , for the third; every revolution meeting upon the cathetus C I.

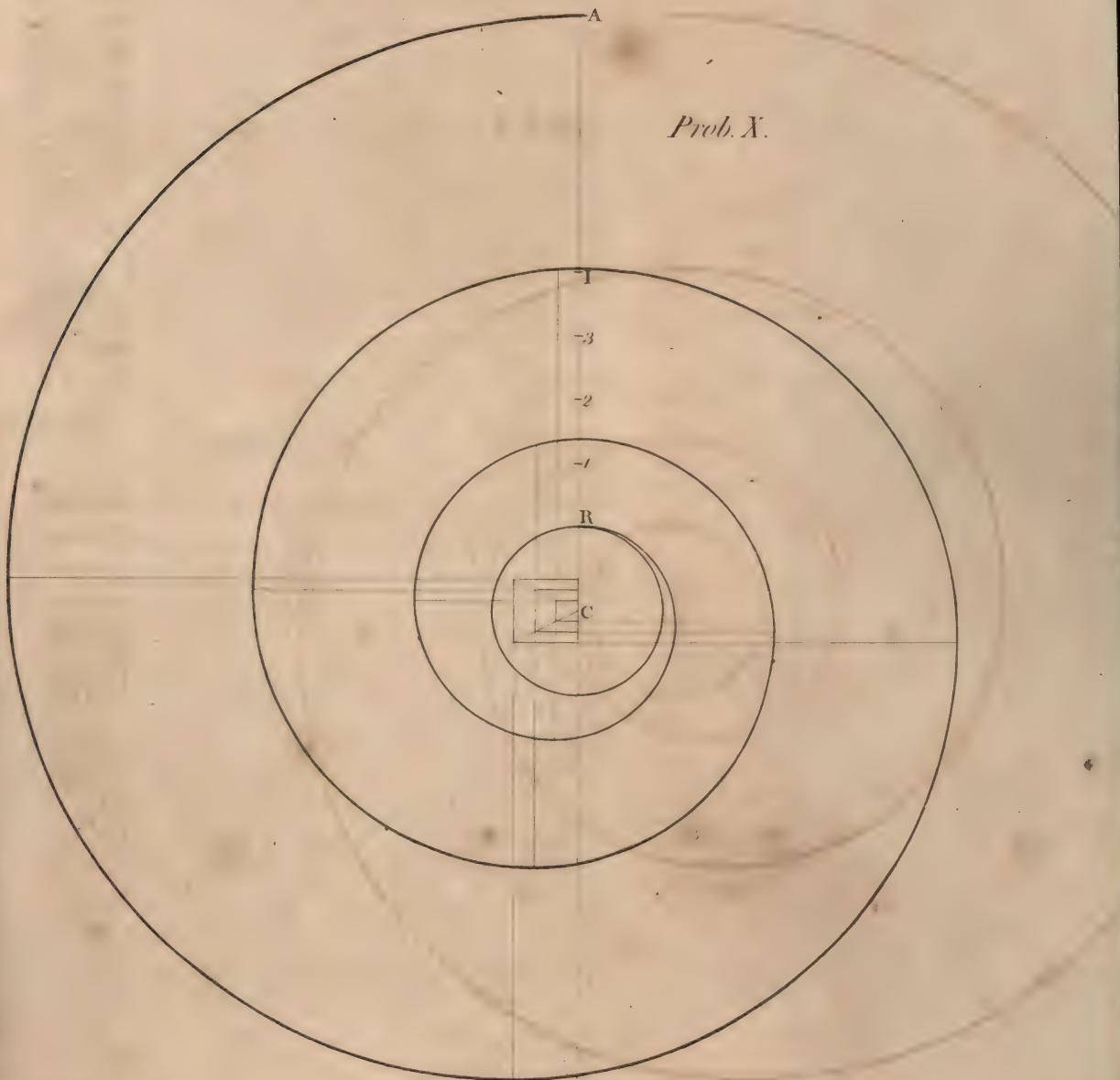
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### PROBLEM IX.

To describe a spiral in the manner of Goldman's volutes, having the height A B to touch a given circle whose diameter is C D, to any number of revolutions.

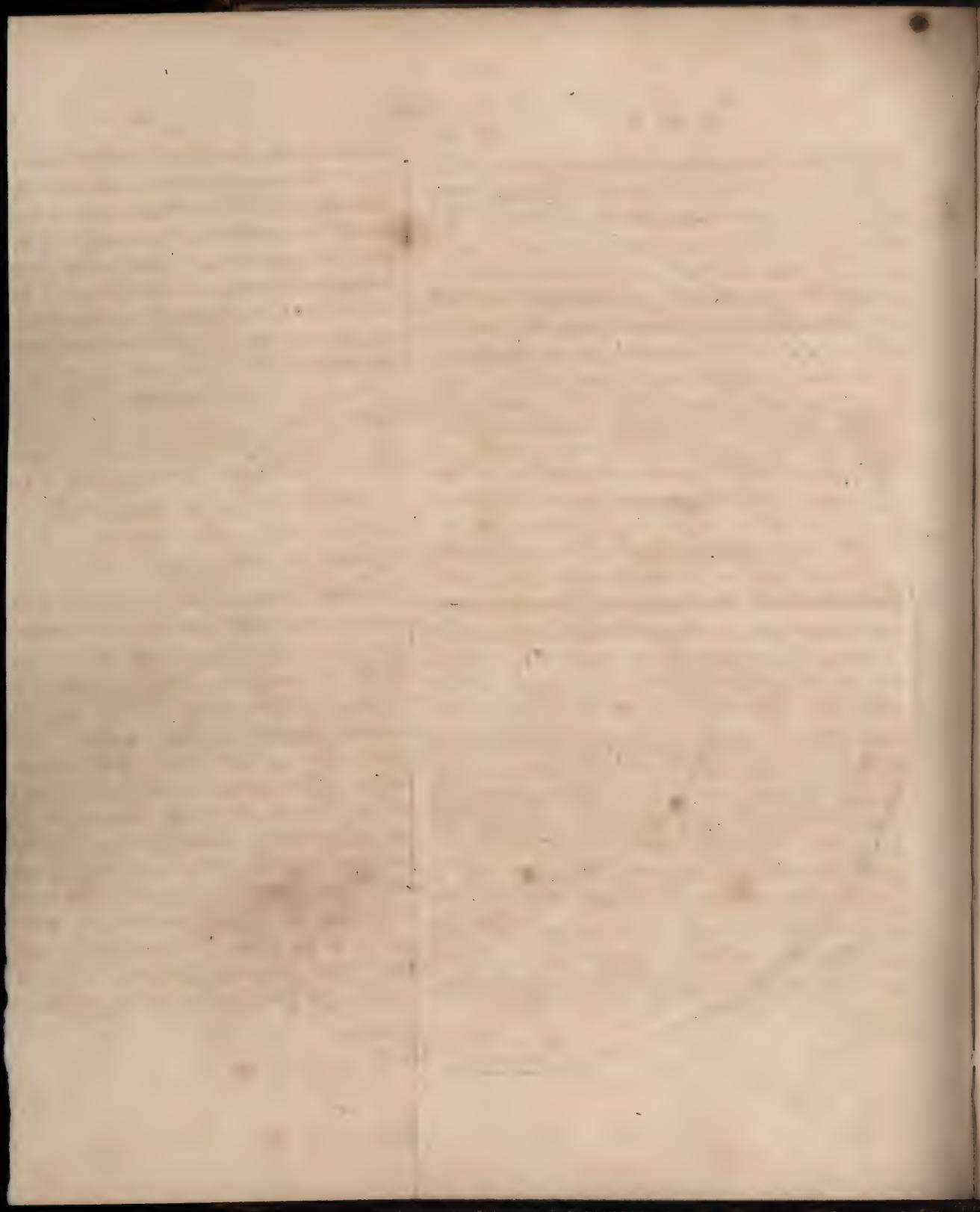
From A B, cut off the part B E equal to the diameter C D of the eye; divide the remainder C A into four times as many equal parts, and two more, as the number of revolutions; from the bottom of the cathetus set up half the number of these parts made less by one part, together with half the diameter of the eye; or from the top of the cathetus set downwards half the number of these parts, and one part more, together with half the diameter of the eye; take one of these parts and make a square, in such a manner that one of the sides may be in the cathetus, and that side must be bisected by the centre of the volute; the square being made upon the right or left side of the cathetus, according as the volute is to be upon the right or left hand. Proceed in every other respect, in what follows, as in the last problem: that is from the centre, draw lines

S P I R A L L I N E S.

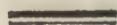


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lines to each of the opposite angles of the square; divide each of those lines into as many equal parts as there are intended to be revolutions; through the points, draw lines parallel to the sides of the square, cutting the cathetus, forming, in all, as many squares as there are revolutions; then the outside square contains the centres of the first revolution; the next square, if more than one revolution, contains the centres of the second revolution; and the third square, if more than two revolutions, contains the centres on its angles of the third revolution; and so on for any other number of revolutions above three, if required.



### PROBLEM X.

The cathetus C A of the spiral, from the centre C being given, to describe it to any number of revolutions to touch a given circle whose centre is C, the centre of the spiral.

On the centre C, with a radius equal to the radius of the eye, describe a circle for the eye; cutting the cathetus C A upwards at R; divide R A into two equal parts at I, and divide R I into one part more than the number of revolutions; that is, if one revolution, it will be divided into two equal parts; and if two revolutions, it will be divided into three equal parts; and if three revolutions, it will be divided into four equal parts, and so on; then take one of these parts for the side of the square, and proceed as in the last problem.

PROB-

## PROBLEM XI.

To draw the spiral of Archimedes with a compass, having the whole height, and the distance between the spiral space.

Divide the given distance between the spiral space, into four equal parts; then take half the whole height, and one of these parts, and set it from the top downwards; or take half the whole height diminished by one part, and set it from the bottom upwards; it will give the centre of the spiral; through which, draw a line at right angles to the height; then construct a square, whose sides are equal to one of the before-mentioned parts, having the centre of the square in the centre of the spiral, and having two of its sides parallel to the perpendicular of the spiral, then the four angles of the square are the four centres; then set the compass on the upper side of the square in that angle which is upon the same side that the spiral is to be drawn, and extend the other leg of the compass to the height, and describe a quadrant; the second centre is the other angle of the upper side of the square; the third directly under, and so on, moving round in the same direction, until there are as many revolutions as are required

## PROJECTION.

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### DEFINITIONS.

1. When straight lines are drawn according to a certain law from the several parts of any figure or object cut by a plane, and by that cutting or intersection describe a figure on that plane, the figure so described is called the projection of the other figure or object.
2. The lines taken all together, which produce the projection of the figure, are called a system of rays.
3. When the system of rays are all parallel to each other, and are cut by a plane perpendicular to them, the projection on the plane is called the orthography of the figure proposed.
4. When the system of parallel rays are perpendicular to the horizon, and projected on a plane parallel to the horizon, the orthographical projection is then called the ichnography, or plan of the figure proposed.
5. When the system of rays are parallel to each other and to the horizon, and if the projection be made on a plane perpendicular to those rays and to the horizon, it is called the elevation of the figure proposed.

In this kind of projections, the projection of any particular point, or line, is sometimes called the seat of that point or line on the plane of projection.

6. If a solid be cut by a plane passing quite through it, the figure of that part of the solid which is cut by the plane, is called a section.

7. When any solid is projected orthographically upon a plane, the outline or boundary of the projection is called the contour or profile of the projection.

*Note.* Although the term orthography signifies, in general, the projection of any plane which is perpendicular to the projecting rays, without regarding the position of the plane on which the object is projected; yet writers on projection substitute it for elevation, as already defined; by which means it will be impossible to know when we mean that particular position of orthographical projection, which is made on a plane perpendicular to the horizon.

### AXIOM.

If any point, line, or plane of any original figure, or object, touch the plane on which it is to be projected, the place where it touches the projecting plane is the projection of that point, line, or plane of the original figure, or object.

### PROPOSITION.

The orthographical projection of a line, which is parallel to the plane of projection, is a line equal and parallel to its original.

### PROB-

## PROBLEM I.

FIG. I.

To project the elevation of a prism standing on a plane perpendicular to the projecting plane; given the base of the prism, and its position to the projecting plane.

Let A B C D, No. 1, be the base of the prism; let H F be the intersection of the projecting plane with the plane on which the prism stands.

Draw lines from every angle of the base, cutting H F at H, and F will be the projection of the points A and C; the angle D, touching H F at D, is its projection.

From each of the points H, D, F, in No. 2, draw the lines H I, D E, and F G, each perpendicular to H F; make D E equal to the height of the prism; through E draw I G, cutting H I and F G at I and G, which will give the projection sought.

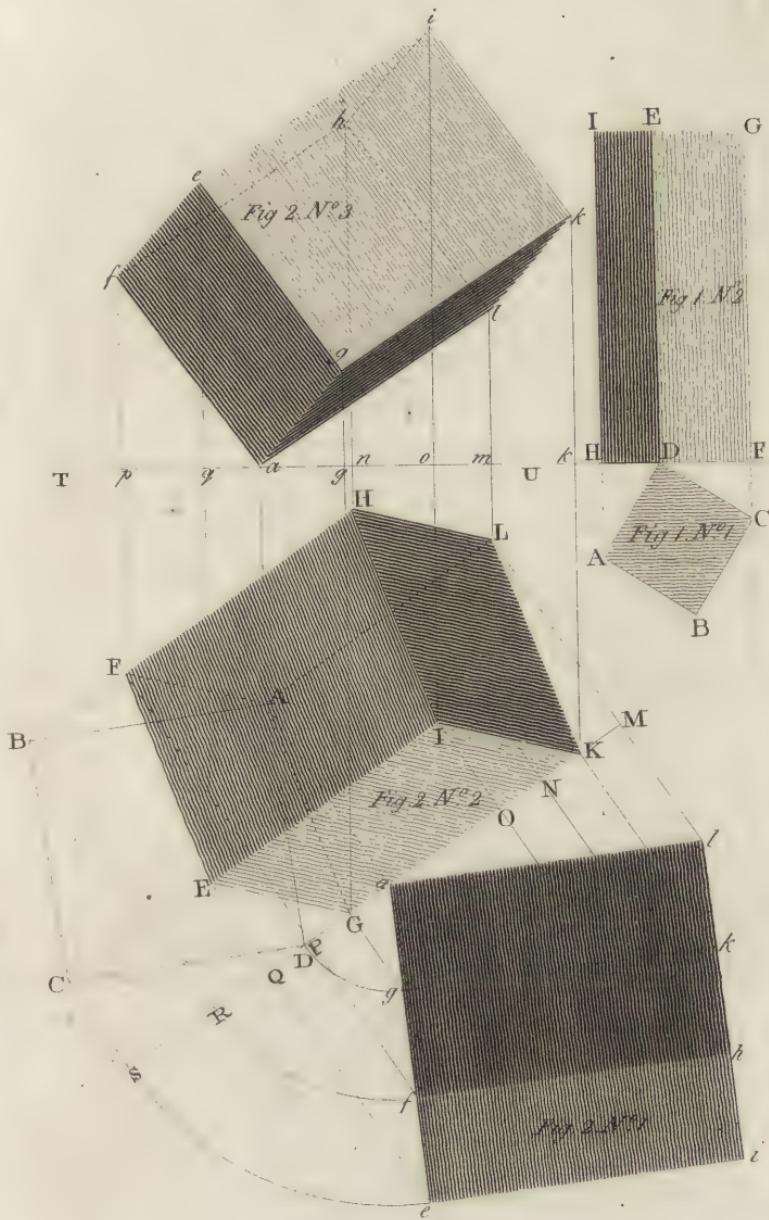
## PROBLEM II.

FIG. 2.

To project the ichnography and elevation of a square prism, to rest upon one of its angles upon a given point A in the plane, on which the ichnography is to be described ; given the ichnography A L of an angle, which the two under planes make with each other; the angle M  $a$  l, which the angle of the solid makes with its ichnography A L; the intersection A  $a$  of one of its ends with the plane of the ichnography; the angle D A  $a$ , which one side of the end makes at A, with the intersection A  $a$  of that end; also given one of the sides of the ends, and the length of the prism.

At the given point A, with the intersection A  $a$ , make an angle  $a$  A D equal to the angle which one of the sides of the end makes with A  $a$ ; make A D equal to one of the sides of the end; then on A D, construct the square A B C D; through the angles of the square B, C, D, draw lines B H, C I, and D M, parallel to A L; then at the point  $a$  in the right line D M, make an angle M  $a$  l with  $a$  M, equal to the angle of the solid, whose projection is A L with A L; make  $a$  l equal to the length of the solid; through the points  $a$  and l, No. 1, draw the lines  $a$  e and  $l$  i, perpendicular to  $a$  l; through the points B and C, No. 2, draw B R and

## PROJECTION





and C S parallel to A a, cutting D M, produced at R and S; on a as a centre with the distances a D, a R, and a S, describe arcs P g, R f, and S e, cutting a e, No. 1, at g, f, e; through the points g, f, e, draw the lines g k, f h, and e i, parallel to a l, and No. 1 will be completed, which will be the projection of the prism on a plane parallel to A L. Through the points g, f, e, draw the lines e E, f F, and g G, perpendicular to D M, or A L, cutting D M, C I, and B H, respectively at G, E, F; also through the points l, k, h, i, draw the lines l L, k K, h H, and i I, likewise parallel to A L, cutting A L, G M, B H, and C I, respectively at the points L, K, H, and I; join E F, E G, and H I, I K, K L, L H, then will the planes E F H I, E I K G, and H I K L, represent the ichnography of the upper sides of the solid; and if F A and A G be joined, then will F A G E, F A L H, and G A L K, represent the sides of the solid next to the plane of projection. Then to project the elevation on a plane whose intersection is T U; from F, E, G, A, H, I, K, and L; that is, from all the points in the ichnography representing the solid angles, draw the lines F f, E e, G g, A a, H h, I i, K k, and L l, perpendicular to the intersection T U, cutting T U at p, q, a, g, o, m, and k; make p f, q e, g g, n h, o i, m l, and k k, at No. 3, respectively equal to P f, Q e, G g, N h, O i, M l, and K k, at No. 1; then join f a, a g, g e, e f; e i, i k, k g, k l, and l a; and f a g e, g e i k, g k l a, will be the elevations of the outside planes of the solid; and by joining f h and h i, f h i e, f h l a, and i h l k will be the elevations of the planes of the solid, next to the plane on which the elevation is projected.

## PROBLEM II.

To project the ichnography and elevation of a leaf; given a section through its middle, and its representation when stretched out on a plane.

Let fig. 1. be the representation of the leaf stretched out on a plane; fig. 2. a section through the middle at A B, fig. 1; and fig. 3. a section of the leaf turned into the true form that the ichnography and elevation is to be projected into; the curve line of fig. 3. being equal in length to fig. 2. that is equal to A B, through the middle of fig. 1.

Divide the curve line of fig. 3. into any number of equal parts, as the points 1, 2, 3, 4, &c.; through these points draw lines perpendicular to the base B A; take any line C D in No. 1 and 2, make all the divisions on C D equal to those of B A, fig. 3, and through these points draw lines at right angles to C D. Also divide A B, fig. 1, into the same number of equal parts as fig. 3 is divided; and through these points draw lines 1 a, 2 b, 3 c, &c. cutting the edge of the leaf at the points a, b, c, d, &c. take all the distances 11 l, 10 k, 9 i, 8 h, 7 g, 6 f, &c. and apply them as ordinates from C D, No. 1 and 2; then the points l, k, i, h, g, &c. will be in the ichnography for No. 1 and 2, and a curve being braced through those points will give half the ichnography or plan; the other half will be found in the same manner: then to find the elevation fig. 4. on a plane parallel to the bottom of the leaf and in fig. 5. making any given angle with the bottom of the leaf, proceed as follows:

Through all the points 1, 2, 3, 4, 5, &c. fig. 3. draw lines parallel to B A, the base of fig. 3; through all the points 12, l, k, i, h, g, &c. in No. 1 and 2, draw lines at right angles to B A in fig. 3. cutting the elevation at the corresponding points 12, l, k, i, h, g, &c. in fig. 4. also at fig. 5. a curve being traced through these points will give the elevation of the leaf.

Fig. 1.

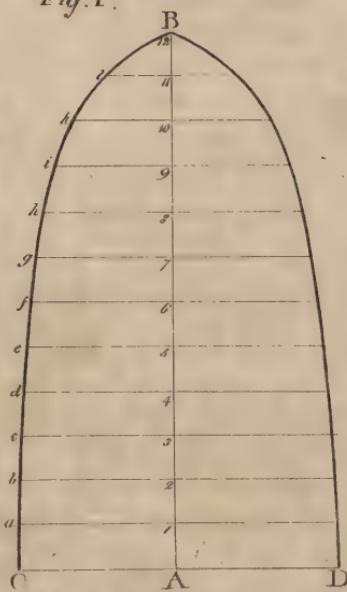


Fig. 2.



Fig. 3.

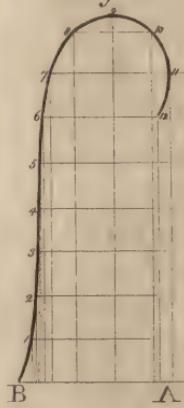


Fig. 4.

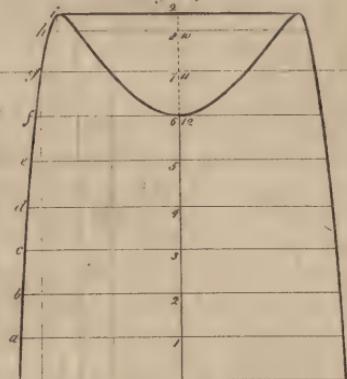
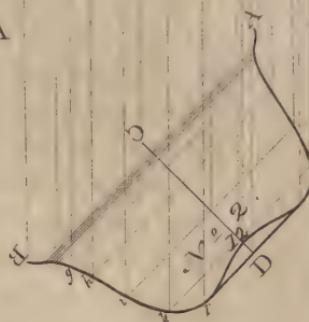
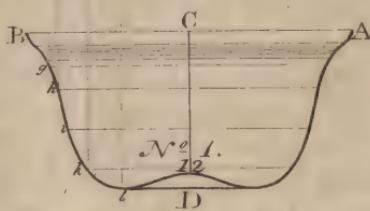
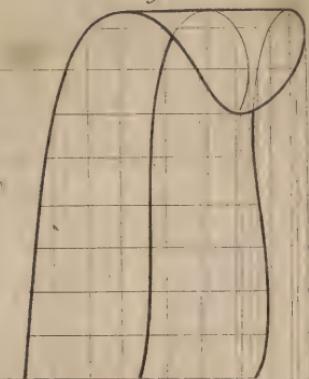


Fig. 5.





THE  
 EFFECT OF DISTANCE  
 ON THE  
 COLOUR OF OBJECTS.

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*DEFINITION.*

The art of giving a due diminution or degradation to the strength of the light and shade and colours of objects, according to the different distances, the quantity of light which fall on their surfaces, and the medium through which they are seen, is called *Keeping*.

1. When objects are removed to a great distance from the eye, the rays of light which they reflect will be less vivid, and the colour will become more diluted, and tinged with a faint bluish cast, by reason of the great body of air through which they are seen.

2. In general, the shadows of objects, according as they are more remote from the eye, will be lighter, and the light parts will become darker ; and at a certain distance the light and shadow are not distinguishable from each other; for both would seem to terminate in a bluish tint, of the colour of the atmosphere, and will appear entirely lost in that colour.

3. If

3. If the rays of light fall upon any coloured substance, the reflected rays will be tinged with the colour of that substance.

4. If the coloured rays be reflected upon any object, the colour of that object will then be compounded of the colour of the reflected rays, and the colour of the object; so that the colour of the object which receives the reflection will be changed into another colour.

5. From the closeness or openness of the place where the object is situated, the light, being much more variously directed, as in objects which are surrounded by buildings, will be more deprived of reflection, and consequently will be darker than those objects which have no other objects in their vicinity; except the surrounding objects are so disposed, as to reflect the rays of light upon the surrounded objects.

6. In a room, the light being more variously directed and reflected than abroad in the open air, (for every aperture gives an inlet to a different stream) which direction is various, according to the place and position of the aperture, whereby every different side of the room, and even the same side in such a situation, will be variously affected with respect to their light, shade, and colours, from what they would in an open place when exposed to rays coming in the same direction.

Some original colours naturally reflect light in a greater proportion than others, though equally exposed to the same degrees of it; whereby their degradation at different

different distances will be different from that of other colours which reflect less light.

The art of keeping a degradation of light and shade on objects, according to their several distances, colours, and other circumstances, is of the utmost consequence to the artist.

In orthographical projections, where equal and similar objects stand in the same position to the plane of projection, they will be represented similar, and of an equal magnitude, at every distance from that plane; and consequently planes which are parallel to each other, would not appear to have any distance; so that the representation of any number of objects, at different distances from each other, would be entirely confused, and no particular object could be distinguished from the others; but by a proper attention to the art of keeping, every object will be distinct and separate, and their respective distances and colours from each other will be preserved; but though a proper degradation of light and shade ought to be preserved according to the respective distances of objects from each other, artists in general take too great liberties with nature; we frequently see in the drawings of architects, the art of keeping carried to so great an extreme, as to render their performances ridiculous.

## SHADOWS.

## DEFINITIONS.

1. A body which is continually emitting a stream of matter from itself, thereby rendering objects visible to our sense of seeing, is called a *luminary*; such as the *sun* or any other body producing the same effect.
2. The stream of matter which is emitted from the luminary, is called *light*.\*
3. A substance or body which light cannot penetrate, is called an *opaque body*.
4. If a space be deprived of light by an opaque body, it is called a *shade*.
5. The whole or part of any surface on which a shade is projected, is called a *shadow*.
6. A body which will admit of light to pass through, is called a *transparent substance*.
7. A line of light emitted from the luminary, is called a *ray*.

\* It will not be expected, that a proper definition of light as a body can be given, seeing it has not as yet been traced to its original cause: some have even doubted its being material; But if so, how comes light to act upon matter, and thereby cause reflection and other effects by it? Before the assertion of its immateriality can be proved, it must first be shown, that nothing will act upon a thing which is absurd:

## PROPOSITION I.

The rays of light after issuing from the luminary proceed in straight lines.

## PROPOSITION II.

If the rays of light fall upon a reflecting plane, the angle made by any incident ray, and a perpendicular to the reflecting plane, is called the angle of incidence, and will be equal to the angle that its reflected ray will make with the same perpendicular called the angle of reflection; these two propositions are known from experiment.

## PROPOSITION III.

If the rays of light fall upon any curved surface, whether concave or convex, or mixed of the two, the angle of reflection will still be equal to the angle of incidence.

## PROPOSITION IV.

Any uneven reflecting surface, whose parts lie in various directions, will reflect the rays of the sun in as many different directions.

## DEMONSTRATION.

If any ray fall upon a part of the surface which is perpendicular to that ray, it will be reflected in the same line as the incident ray; but the more or less any part of the surface is inclined to a ray falling upon that part of the surface, the greater or less angle will the reflected ray make with the incident ray. For, imagine a perpendicular to be erected to that part of the surface where any incident ray impinges on the surface, it is evident that the measure of the angle of incidence is equal to the obtuse angle made by the incident ray, and the reflecting surface at the impinging

point, made less by a right angle; but the angle of reflection is equal to the angle of incidence; wherefore it follows that the whole angle formed by the incident and reflected rays, are double of the angle of incidence; and consequently a reflecting surface, whose parts lie in various directions, will reflect the sun's rays in as many directions.

*Corollary.* Hence appears the reason why objects and their parts become visible to our sight when immersed in shade.

**DEFINITIONS.**

1. If a given straight line pass through or cut a given plane, and if an imaginary plane be supposed to pass through any two points in the straight line, perpendicular to the other plane, the angle made by the intersection of the two planes, and the given straight line, is called the inclination or altitude of the given line, on the given plane.

2. The intersection of the two planes, is called the seat of the given line on the given plane.

*Corollary.* The angle made by a ray of light, and the seat of that ray, is the angle of the sun's inclination.

3. If on the surface of any solid, there be any point or points in the surface where the sun's rays fall perpendicular, this point or points which the sun's rays fall perpendicular to, are called points of light.

4. If on the surface of any solid, there be any line drawn upon that surface, and if the line so drawn upon the surface be lighter than any other line that can be drawn upon the said surface; then the line first drawn is called the line of light.

5. If the sun's rays fall upon any solid, and if a line or lines be drawn on the surface of the solid where the sun's rays are a tangent, or upon the place or places of the surface which divide the dark side from the light side; then the line, or lines, so described, are called a line or lines of shade.

6. If the sun's rays be parallel to any plane, that plane to which they are parallel is called a plane of shade.

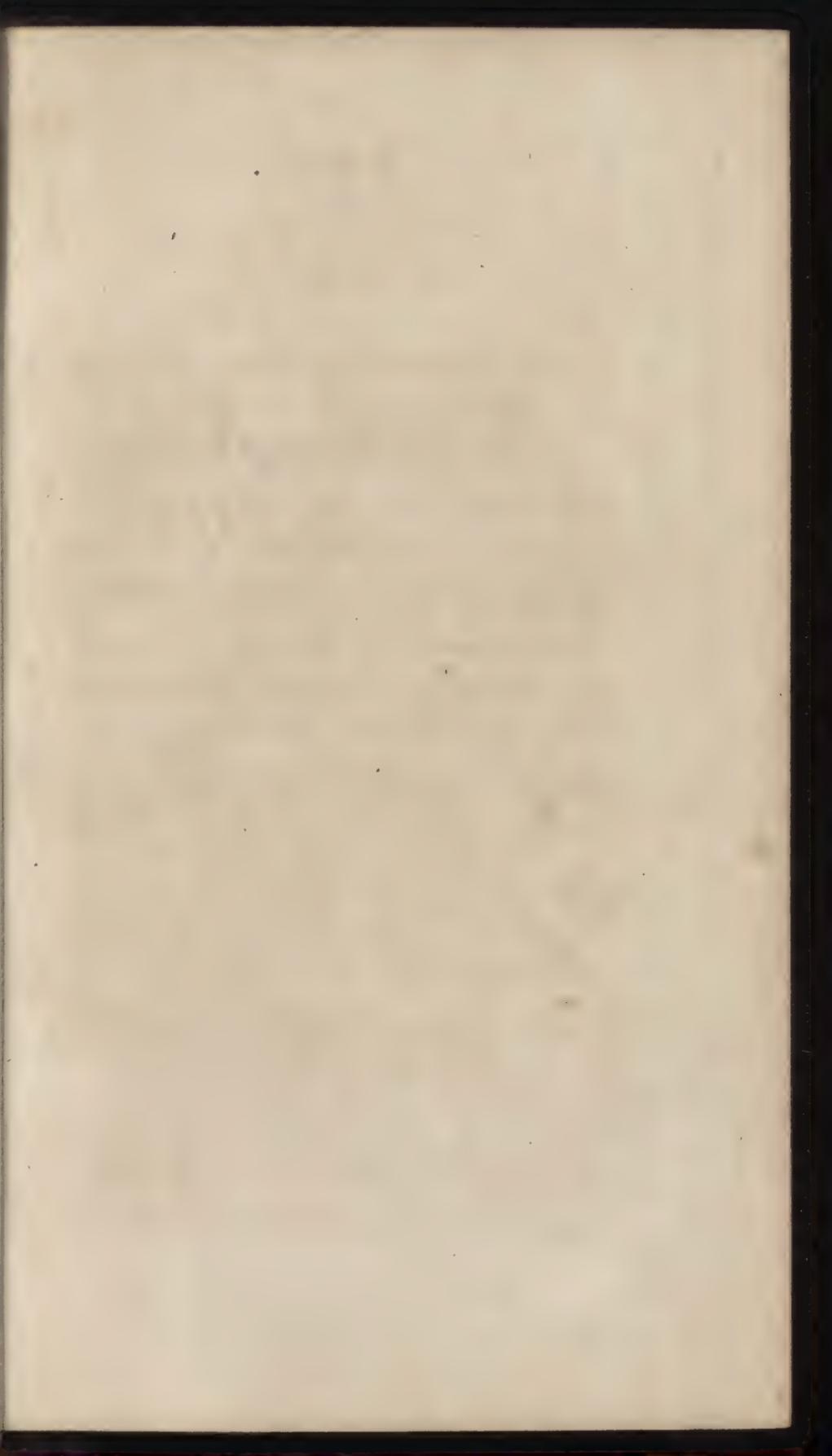
PROB-

## PROBLEM I.

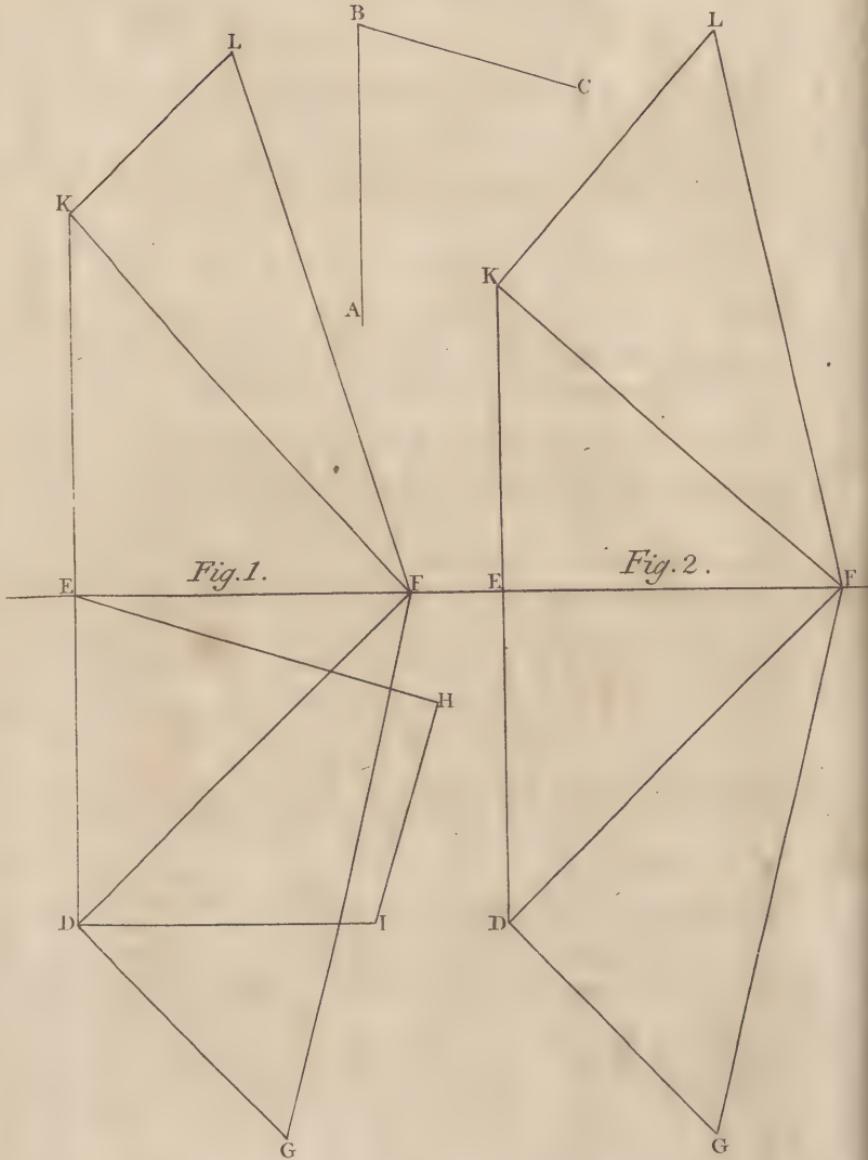
Given the ichnography and elevation of a prism, whose sides stand perpendicular to the horizon, and whose ichnography is a figure of any kind, regular or irregular; given the seat of the sun on the ichnography, also on the elevation; and the intersection of the plane of the elevation, with the plane of the ichnography; the representation of the point being likewise given on the elevation, and also on the ichnography, to determine the representation of the shadow on the elevation.

Through the representation of the given point in the plane of the ichnography, draw a line parallel to the seat of the sun's rays on that plane, and produce it till it cut the intersection; from that point on the elevation, draw a line perpendicular to the intersection; then through the representation of the given point on the elevation, draw a line parallel to the sun's seat on the elevation, cutting the line that was drawn perpendicular to the intersection, and that point will be the representation of the shadow on the elevation.

PROB-



## SHADOWS



## PROBLEM II.

PLATE 79. FIG. 1.

Given the altitude and seat of the sun on the horizon, and the intersection of a plane, making a given angle with the horizon; to find the seat and altitude of the sun on the other plane.

Let D F be the seat of the sun on the horizon, and D F G the angle of the sun's elevation, E F the intersection of the plane with the horizon, and A B C the angle which that plane is to make with the horizon.

Produce D F till it meet E F in F; through any point D, in the seat D F, draw D G perpendicular to D F, cutting F G in G; also through D draw D K perpendicular to E F, cutting E F in E; through D draw D I perpendicular to D K; make the angle D E H equal to the given angle A B C; make D I equal to D G; through I, draw I H perpendicular to E H, cutting it in H; make E K equal to E H; join K F; from K make K L perpendicular to K F; from K make K L equal to H I; join L F; then will K I be the seat of the sun on the other plane, and K F L will be the angle of the altitude.

If the plane K E F stands perpendicular to the horizon, as in fig. 2. the operation will be more simple, as follows, the same letters standing for the same things:

Make E K equal to D G; join K F; draw K L as before, and make K L equal to D E; join L F; then will K F be the seat of the sun on the horizon, and K F L be the seat of the altitude.

PROB.

## PROBLEM III.

PLATE 80. FIG. 1.

Given the ichnography A B C D E F G H I K, and elevation L M N O of an upright prism, whose base or ichnography is a regular polygon, and the seat of the sun's rays on the base; to determine the various degrees of light and shade on the different sides of the prism.

Let P Q in the ichnography be the seat of the sun; and if it cut C D perpendicular, then will C D be the lightest side of the prism, the sides D E and C B will be a small degree darker, as P Q is more inclined to D E and C B; and in general, according as the sides recede on each side of C D, they will be continually darker, until they become wholly deprived of light; then suppose the sun's rays to touch the side G H, then G H will be the plane of shade, or that side where the light will end.

Much in the same manner may the different degrees of shade be found on the surface of a cylinder, as in fig. 2. where A B C D is the ichnography, and G H I K the elevation: that is, if B P be the direction of the sun's rays, cutting the ichnography in B, then will B be the lightest place; and it will be continually darker and darker in each side of the point B, until it arrives at the point C, where the ray touches the side of the cylinder; and there the light will end, and the darkness begin.

PROB-

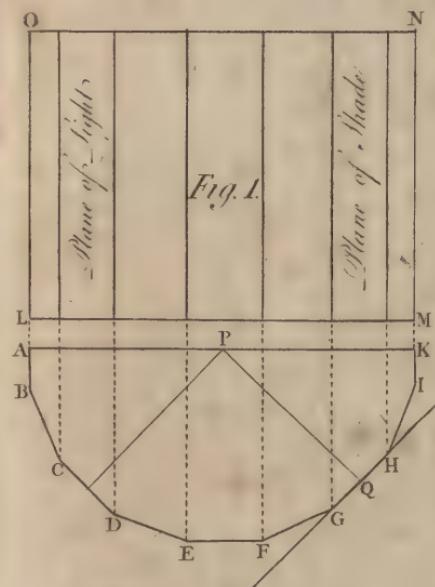


Fig. 1

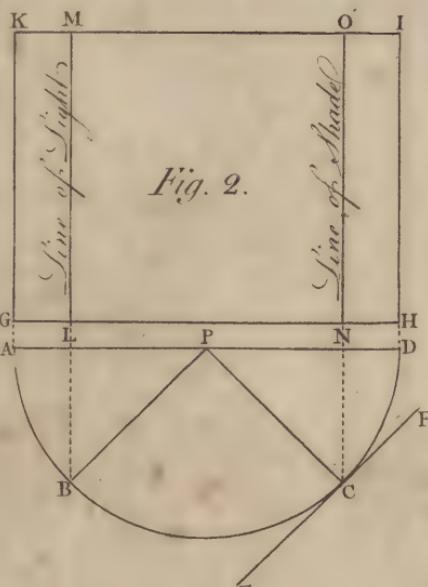


Fig. 2.

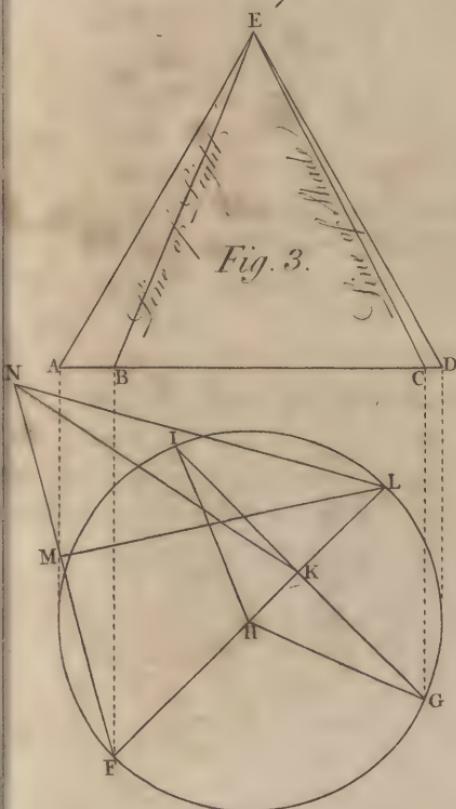


Fig. 3.

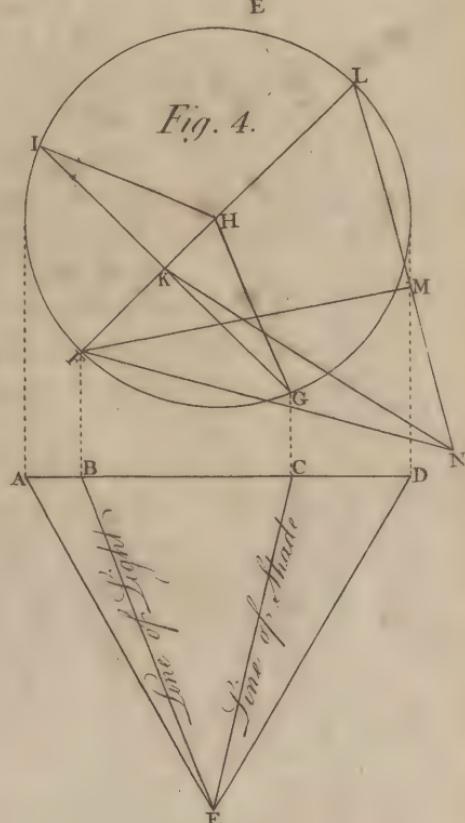


Fig. 4.



## PROBLEM IV.

FIG. 3. and 4.

To represent the boundaries of light and shade on the elevation of a cone illumined by the sun, given the angle that any ray of light makes with the base of the cone; also to determine the line of light, or that place on the surface that will be the lightest.

Let  $A E D$  be the elevation of the cone, and let  $F I L G$  be the ichnography or base of the cone; let  $F L$  be the seat of any ray in a plane, passing through the axis, and let the angles  $M L F$ , fig. 3. and  $L F M$  fig. 4. be the angles which a ray of light makes with the base of the cone, and let  $F N L$  be a section of the cone passing through the axis, and through  $F L$ ; consequently the rays  $M L$  and  $F M$ , will be in that plane.

Produce the rays  $M L$  and  $F M$ , if necessary, until they cut the sides of the section  $N F$  and  $N L$ ; fig. 3. will be cut at the points  $M$  and  $L$ , and fig. 4. at the points  $F$  and  $M$ : bisect each of the lines  $M L$  and  $F M$ ; and through  $N$ , the vertex of the cone, and the point of bisection, draw  $N K$ ; through  $K$ , draw  $I K G$  at right angles to  $F L$ ; through the points  $G$  and  $F$ , draw the lines  $G C$  and  $F B$ , perpendicular to  $A D$ , the representation of the base, cutting  $A D$  at  $B$  and  $C$  in the elevation, and join  $B E$  and  $C E$ : then will  $B E$  be the lightest line that can be drawn on the surface of the cone, and  $C E$  will be the representation of a line which will divide the light side of the cone from the dark side, and  $B E C$  will be the representation of half the enlightened side of the cone.

## PROBLEM V.

## PLATE 81.

Given the ichnography and elevation of a polygonal ring, or a ring made of cylinders of equal lengths, and making equal angles with each other, to determine the representation of the boundaries of light and shade, on the ichnography and elevation; the sun's altitude, and seat to the plane on which the ichnography is described, being given.

Let **A C** be the seat of the sun in the plane of the ichnography, cutting the thickness of the ring at **A** and **B**; let **A C D** be the angle which the sun's rays make with the plane of the ichnography, or seat **A C**.

Bisect **A B** in **c**; with the radius **c A** or **c B**, describe a circle; and through the centre **c**, draw **c d** parallel to **C D**, cutting the circle at **d**; also through **e**, draw the line **a b**, at right angles to **c d**, cutting the circle at **a** and **b**; through the points **d** and **b**, draw lines parallel to the sides **A** and **B** of the ring; then the dark line nearest to **A**, will represent the line of light; and that which is nearest to **B**, will represent that line which separates the light side from the dark side:

To find the lines of light and shade on the next side of the ichnography: from the centre **C**, draw **C E** at right angles to the side **E**, cutting the sides **E** and **F** at **E** and **F**; through the point **A**, draw **H A G** at right angles

## SHADOWS.

Fig. 2.

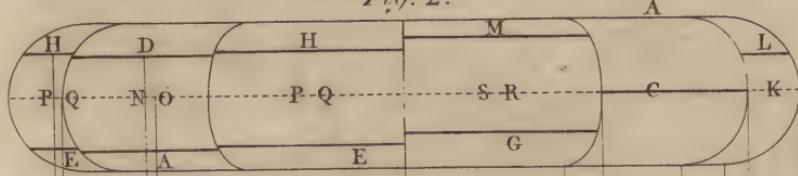
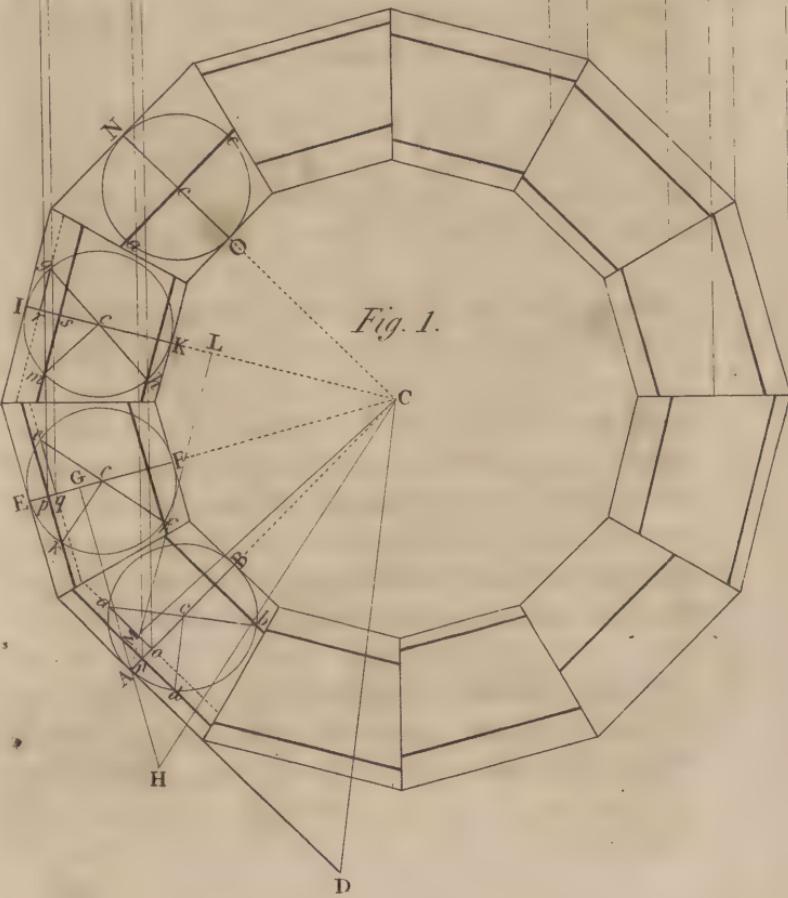


Fig. 1.





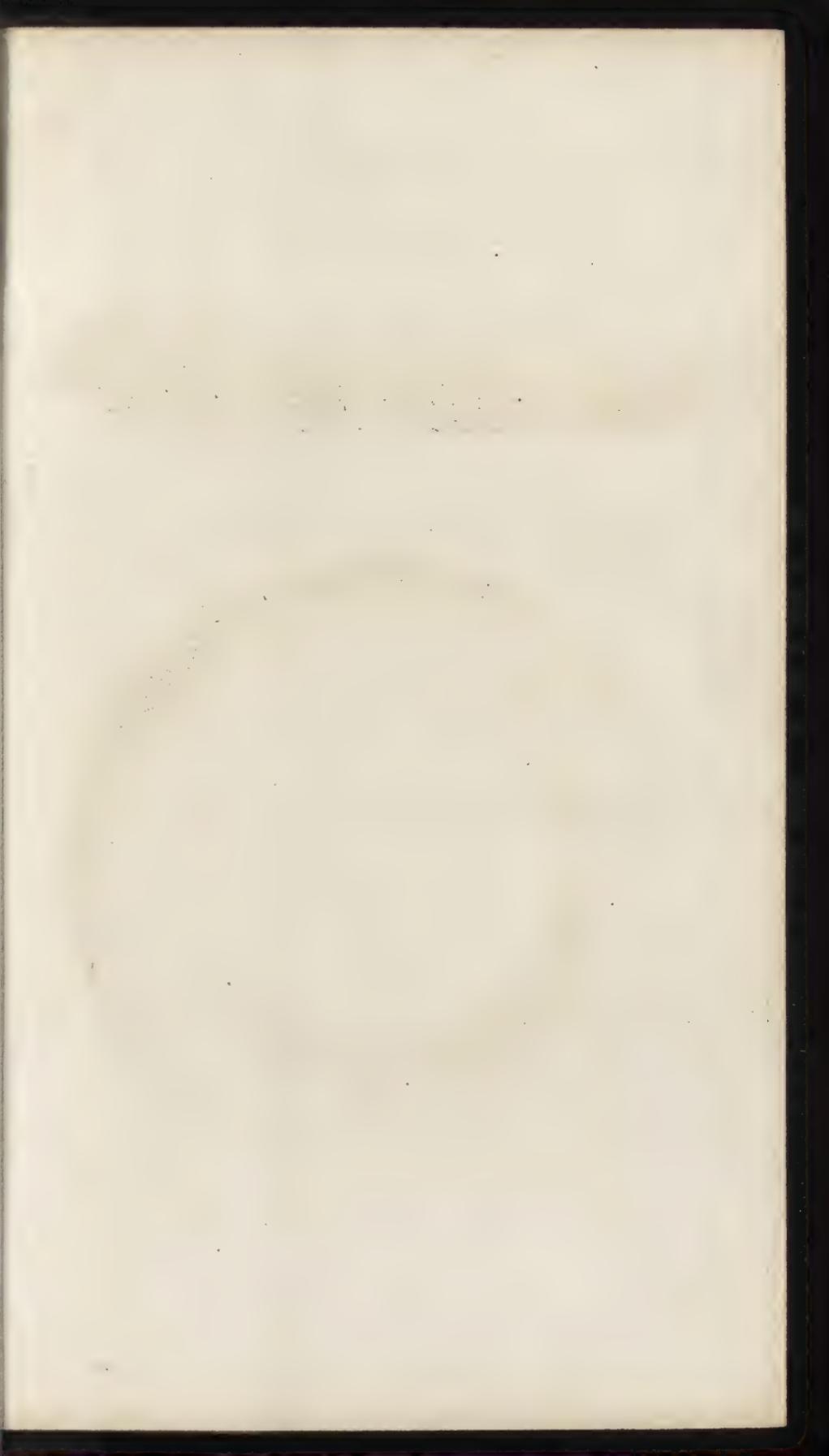
angles to  $E C$ , cutting  $E C$  at  $G$ ; from  $G$  make  $G H$  equal to  $A D$ ; join  $H C$ , and bisect  $E F$  at  $c$ : then with the radius  $c E$ , on  $c F$ , describe a circle, and through its centre  $c$ , draw  $c k$  parallel to  $C H$ , cutting the circle at  $k$ ; also through  $c$  draw  $p f$  at right angles to  $c k$ ; through the points  $k$  and  $f$ , draw lines parallel to the sides  $E$  and  $F$ : then will the line next to  $E$ , that cuts  $E F$  at  $p$ , be the line of light, and the line next to  $F$ , the line of shade.

In like manner proceed with the sides  $I$  and  $K$ : that is, through  $C$ , draw the line  $C I$ , at right angles to the side  $I$ , cutting the sides  $I$  and  $K$ , at  $I$  and  $K$ ; bisect  $I K$  at  $c$ ; with the radius  $c I$  or  $c K$ , describe a circle; through the point  $A$ , as before, draw the line  $A L$  perpendicular to  $C I$ , cutting  $C I$  at  $L$ ; from  $L$ , make  $L M$  equal to  $A D$ ; join  $M C$ ; through  $c$ , draw  $c m$  parallel to  $C M$ , cutting the circle at  $m$ ; also through  $c$ , draw  $g h$  perpendicular to  $c m$ , cutting the circle at  $g$  and  $h$ ; then through the points  $m$  and  $h$ , draw lines parallel to the sides  $I$  and  $K$ : then the line next to  $I$ , which cuts  $I K$  at  $s$ , will give the line of light, and the line next to  $K$  the line of shade.

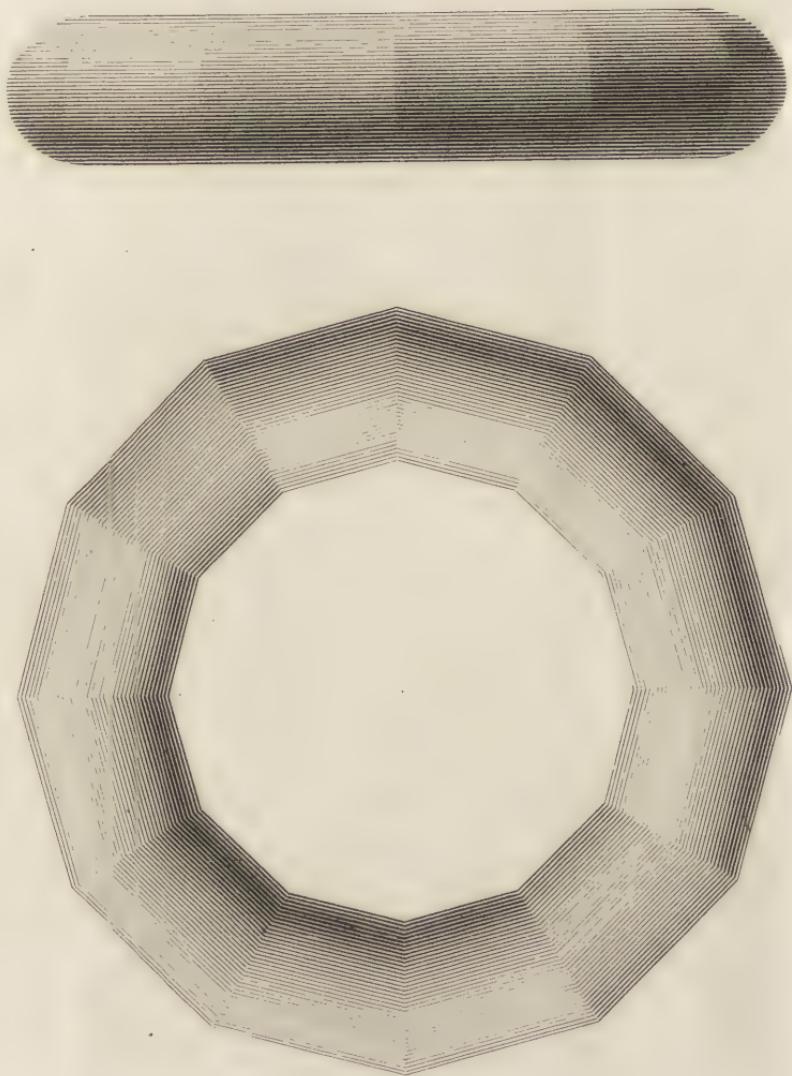
In like manner to find the common boundary of light and shade, upon that side of the ring next to the ichnography, draw lines through the points  $a$ ,  $p$ ,  $g$ , parallel to the sides  $A$ ,  $E$ ,  $I$ , as are shown by the dotted lines, which will give lines of shade on the under side of each cylindrical part. The lines for one quarter of the ring being found, the other quarter, on the other side of the seat  $A C$ , may be found from the last quarter, each being in the same order, receding from the line  $A C$ , or drawing towards it. One half being now found the

other half will be found by observing that opposite sides of the ring are parallel to each other ; and consequently if one is found, the other will also be found ; for the light will be at the same distance upon the same sides of that which is to be found, as that cylinder which is found. Then to find the lightest lines on the elevation fig. 2, and also those lines which will be the boundaries of the light and shade, proceed as follows :

Through the points *n*, *o*, *p*, *q*, draw the lines *n N*, *o O*, *p P*, *q Q*, perpendicular to the elevation, cutting the line *P K*, which represents a plane passing through the axis of all the straight cylinders, at the points *P*, *Q*, *N*, *O*; make *P H* equal to *p k* ; from *Q*, make *Q E* equal to *q e* ; from *N*, make *N D* equal to *n d* ; from *O*, make *O A* equal to *o a* ; then through the points *H*, *D*, *E*, *A*, fig. 2, draw lines parallel to *P K* : then will the lines *H* and *D*, represent the lines of light, and *E* and *A* will represent the lines of shade. Now since the side **D A**, in the elevation fig. 2, represents the cylindrical part **A B** on the ichnography, and because that the lines of light and shade are in the same order on each side of **A B** : that is, the light and shade will be the same height from the plane of the ichnography, on each of the cylindrical parts that are equidistant from the cylindrical part **A B**, on each side of it, and consequently will be represented on each of the cylindrical parts fig. 2, equidistant from **A D**, at the same altitude ; then make *P H* and *Q E*, the next cylindrical part to the centre of the elevation, equal to *P H* and *Q E*, on the outside cylindrical part, which the side *E F* on the ichnography represents ; make *S M* and *R G*, in the elevation, equal to *s m* and *r g*, on the ichnography ; the height of the lines on each side of the elevation representing



## SHADOWS



Drawn by P. Nicholson

Engraved by M. Lowry

London, Published June 1, 1795, by P. Nicholson & C°

presenting the light and shade, being taken from its corresponding place on the ichnography, as already shown, will complete the lines of light and shade on the elevation. This preparation is necessary to the shading of plate 82,

### OBSERVATIONS.

1. If the seat of the sun's rays be drawn on any plane, and if a cylinder lay on that plane with its axis perpendicular to its seat, and parallel to the plane, the lightest line on the cylinder will be nearer the plane in this position than in any other. 2. But if the axis of the cylinder make oblique angles, then the line of light will be higher on the cylinder. 3. Again, if the axis of the cylinder be parallel to the seat of the sun, the lightest line on the cylinder, in this case, will be at its greatest distance from the plane, because then the line of light will be where a plane passing through the axis of the cylinder cuts the upper surface perpendicular to the plane on which the cylinder lies; the line of light in the first position of the cylinder, is brighter than the line of light in the second position of the cylinder, and the line of light in the second position of the cylinder is brighter than the line of light in the third position of the cylinder; the axis of the cylinder in all these cases being parallel and equidistant from the plane of the ichnography: consequently the lines of light in the first and third positions of the cylinder, are the extremes of all the varieties which happen between these two positions: that is, the first is the lightest possible, and the third is the darkest possible. The boundaries of light and shade, called in this book the lines of shade, or those lines which separate the

light

light side from the dark side, are always at a quadrant's distance from each other ; and consequently that arc which is the under line of darkness in the first position, will be higher in the second position, and still higher in the third position ; and if a plane be made to touch the cylinder upon that line, it will be perpendicular to the plane on which the cylinder lies, and therefore its ichnography will be represented by the edge of the cylinder. Now suppose the cylinder to be turned round by moving continually the same way, until the axes of the cylinder come into that position in which the axis would be perpendicular with its seat, then the line of darkness would be in its highest position.

The example of this polygonal ring, is to show the manner of shadowing cylinders, in different positions, when their axes are parallel to the same plane ; this will be more clear, by comparing plate 81 with 82.

---

### PROBLEM V.

To find the lightest line, also the line that divide the lightest side from the dark side on the ichnography and elevation of a circular ring.

Let A C be the seat of the sun, or any line parallel to it passing through the centre C of the ichnography fig. 1, cutting the thickness of the ring at B and H ; and let B C D be the sun's altitude ; take the point G, at one quarter of the circumference of the ring, distant from the point B ; and take any points E and F in the circumference between B and G, and draw the lines E C, F C, and G C, cutting the inside of the ring at I, K, and



## SHADOWS

Fig. 2.

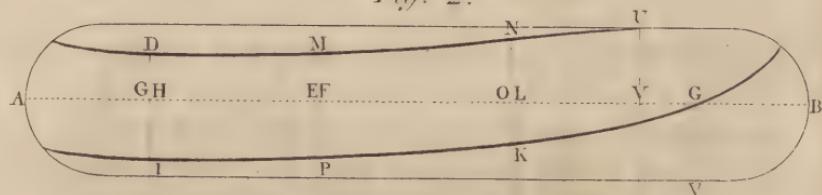
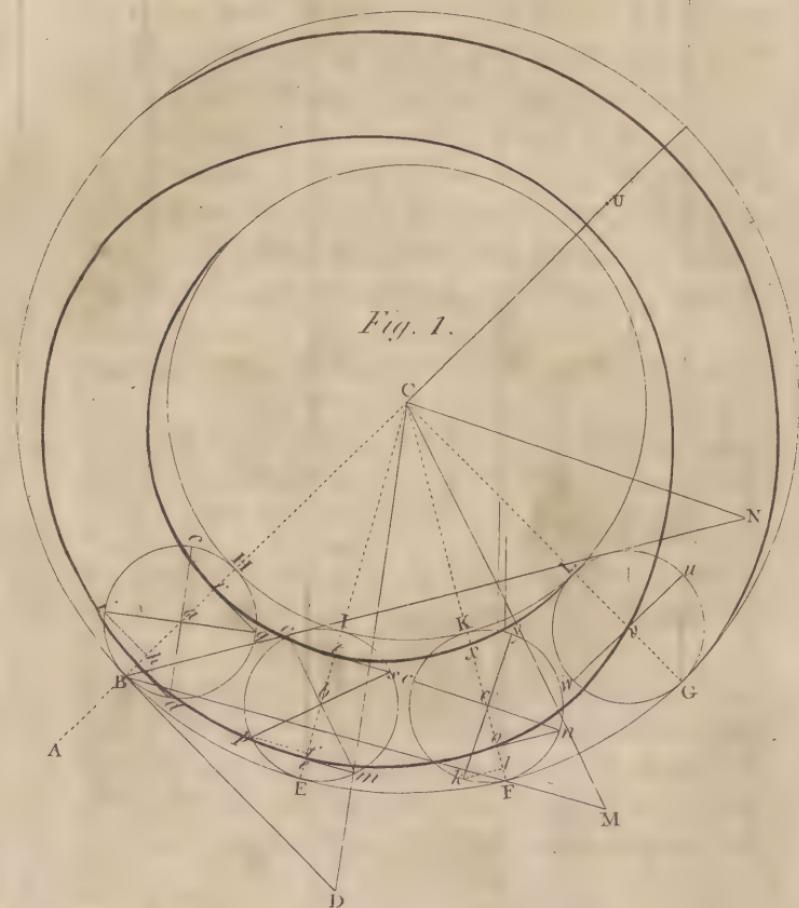


Fig. 1.



and L; through the point B, draw the lines B M, B N, and B U, each respectively perpendicular to E C, F C, and G C, cutting E C, F C, and G C, at O and P; make O M, P N, and C U, each equal to B D; join C M, C N, and C U, on B H, E I, F K, and G L, as diameters, describe circles, whose centres are *a*, *b*, *c*, *a*; through these centres draw the lines *c d*, *c m*, *c n*, and *w u*, parallel to C D, C M, C N, and C U, cutting the circles at *a*, *m*, *n*, *u*, and *c*, *c*, *c*, *w*; also through the centres *a*, *b*, *c*, *v*, draw lines *i q*, *p s*, *k y*, and L G, respectively perpendicular to C D, C M, C N, and C U, cutting the circle at *i*, *q*, *p*, *s*, *k*, *y*, L, and G.

Through the points *d*, *m*, *n*, *u*, draw the lines *d g*, *m e*, *n o*, and *u v*, perpendicular to the diameters B H, E I, F K, and G L, cutting B H, E I, F K, and G L, respectively at the points *g*, *e*, *o*, *v*; draw a curve which will be part of the line of light for one quarter; also through the points *q*, *s*, *y*, draw the lines *q r*, *s t*, and *y x*, cutting the diameters at *r*, *t*, *x*; then through the points *r*, *t*, *x*, and L, draw a curve line *r t x L*, which will be the upper line of shade for that quarter, or that line which divides the dark side from the light side, upon the upper side of the ring on that half which is next to the luminary.

One quarter being now found of the line of light and shade, on the ichnography of the visible side of the ring, the other three quarters will be found from that quarter which is already found, in the same manner as in the last problem, and the points D, M, N, U, also I, P, K, G, in the elevation; the curves D, M, N, U, and I, P, K, G, being drawn, then D M N U will be the line of light, and I P K G the line of shade.

*Observations*

*Observations for the shadowed Plate.*

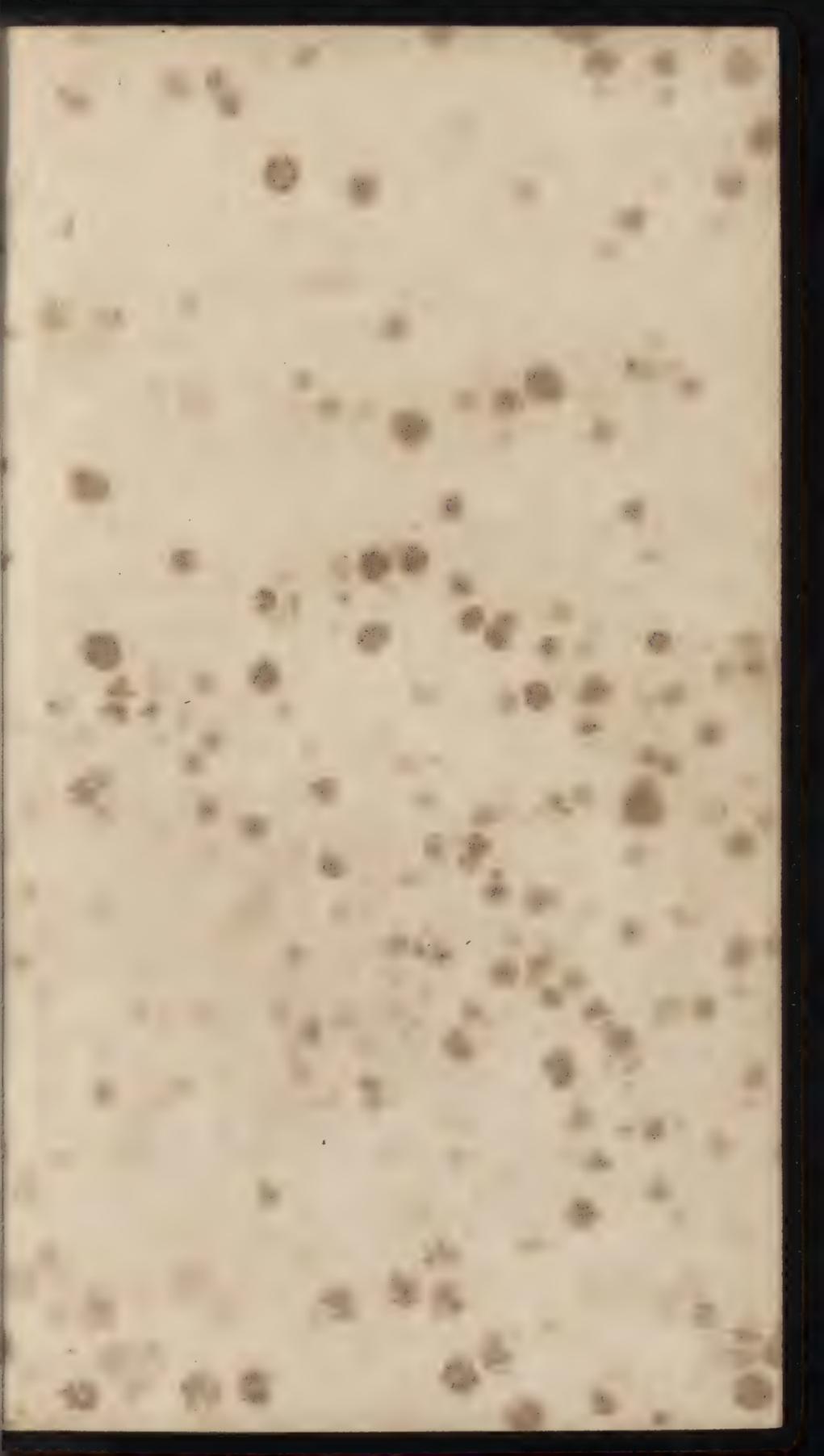
In the ichnography, fig. 1. the points *g* and *Q* in the line of light, which are diametrically opposite to each other, are lighter than any other point or points in that line, because the sun's rays strike the ring perpendicular at these two points in that line, but at no other ; and the points *e*, *o*, *v*, as they recede from *g*, will be gradually darker and darker : then the line of light on each side of the point *g* will be darker and darker, until it arrives at a quadrant's distance on each side of it : that is, at *v* and *R*: then the points *v* and *R*, will be darker than any other point or points taken in the line of light ; and the points *R* and *v*, which represent the highest points of the ring, will divide the thickness of the ichnography into two equal parts at these points.

In the line of shade, namely, that line which is most distant from the plane of the ichnography, which will be the upper line of shade, on that half of the ring which is next to the luminary, at the point *r*, it will be highest ; and as it recedes on each side of the point *r*, it will be lower and lower, until it arrives to the opposite side of the ring, which will be the lowest point upon the sides at *S* and *L*; at a quadrant's distance from *H*, it will touch the inner ichnography at *S* and *L*; that is, at half the height of the ring from the plane of the ichnography.

# SHADOWS

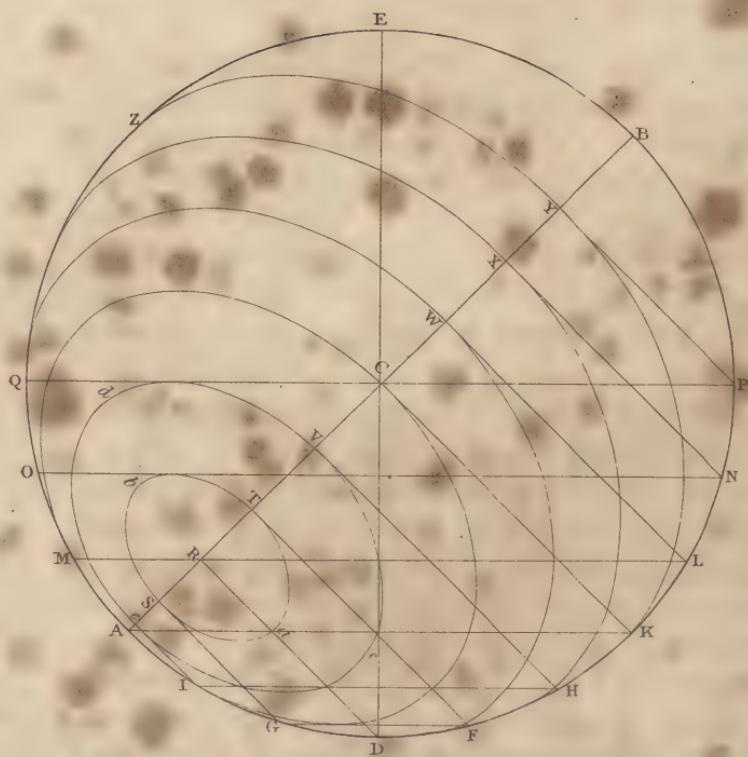






1.85. \*

# SHADOWS.



by P. Nicholson

Engraved by J.

## PROBLEM VII.\*

## PLATE 85.\*

To represent the lines of equal gradation on the surface of a sphere given; the seat of a ray of light on the plane of projection, and the elevation of the ray.

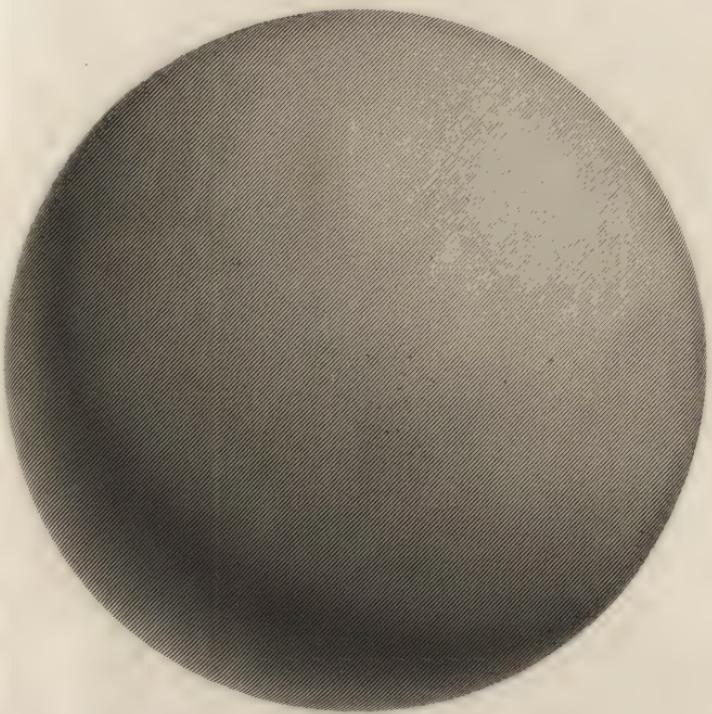
Let  $A B$  be the diameter of the sphere and seat of the sun. Find the centre  $C$ , and describe a circle with the radius  $C A$  or  $C B$ ; this circle will represent the contour of the sphere. Make the angle  $A C D$  equal the elevation of the ray above the seat  $C A$ ; and let  $C D$  be produced so as to cut the circumferent line in  $D$  and  $E$ . Draw the lines  $F G$ ,  $H I$ ,  $K A$ ,  $L M$ ,  $N O$ ,  $P Q$ , perpendicular to  $D E$ , to cut the circle at the points  $F$ ,  $G$ ,  $H$ ,  $I$ ,  $K$ ,  $A$ ,  $L$ ,  $M$ ,  $N$ ,  $O$ ,  $P$ ,  $Q$ , then will  $F G$ ,  $H I$ ,  $K A$ ,  $L M$ ,  $N O$ ,  $P Q$ , be the diameters of circles which have equal intensities of light around each circumference on the sphere. Draw  $D R$ ,  $G S$ ,  $F T$ ,  $H V$ ,  $K C$ ,  $L W$ ,  $N X$ ,  $P Y$ , perpendicular to  $B A$ ; then  $R$  will be the projected point, representing the lightest point on the surface;  $T S$  is the shorter axis of the ellipsis, and  $F G$  the greater. Describe the ellipsis  $a T b s$ , which will represent a circle of equal intensity of light in every part of its circumference on the surface of the sphere. In like manner, if the ellipsis  $c V d e$  be drawn, it will represent another circle of equal intensity. Proceed in this manner, and represent all the intermediate circles of the sphere to that, the diameter of

which is **P Q**, where a ray of the sun would at any point be a tangent, and the representation of this last circle **K Y Z** will be the line of separation of light and shade; then every succeeding ellipsis toward the lightest point **R** will represent graduating lines continually lighter.

PLATE 86.\*

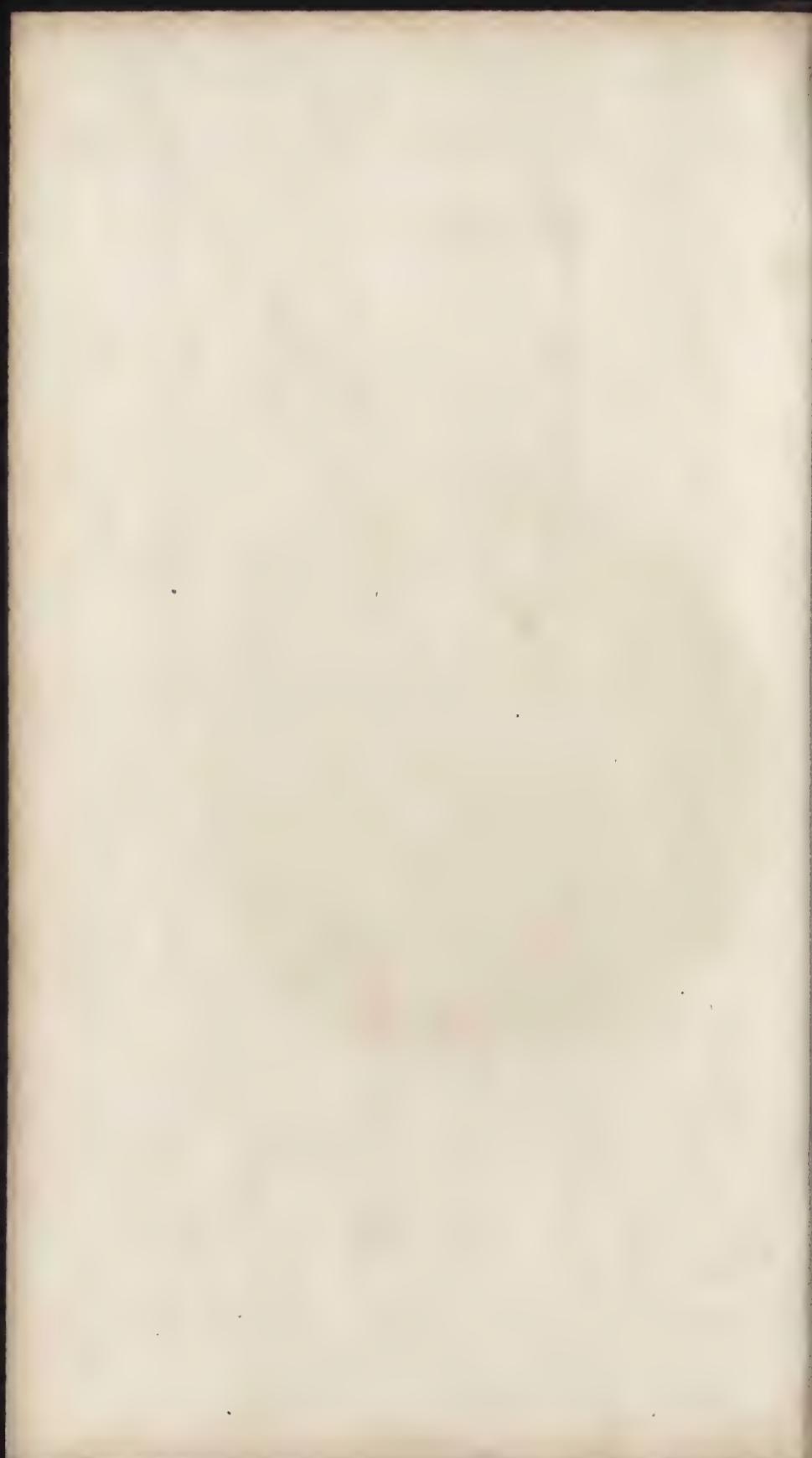
Shadowed according to the former projection.

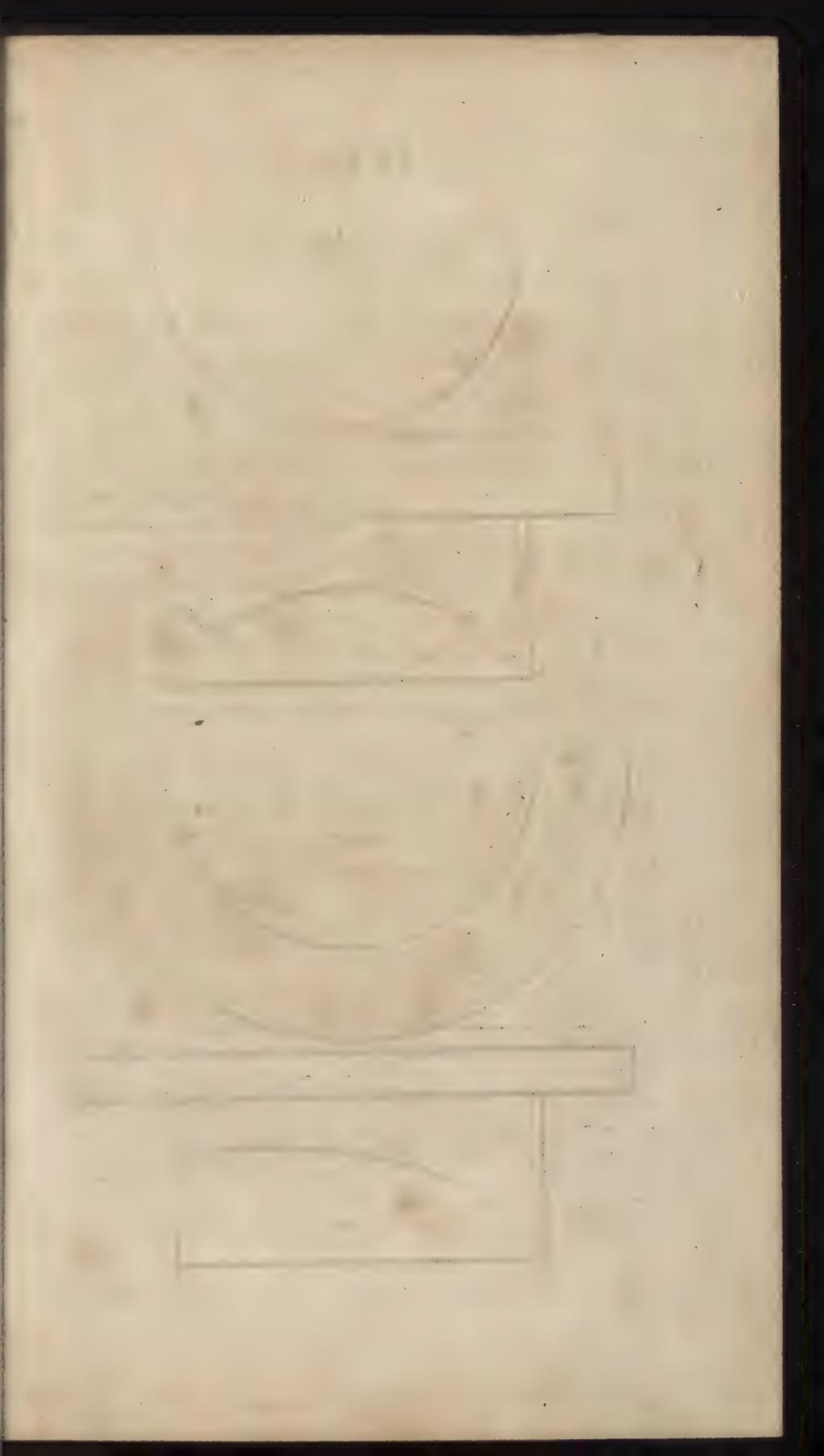
SHADOWS.



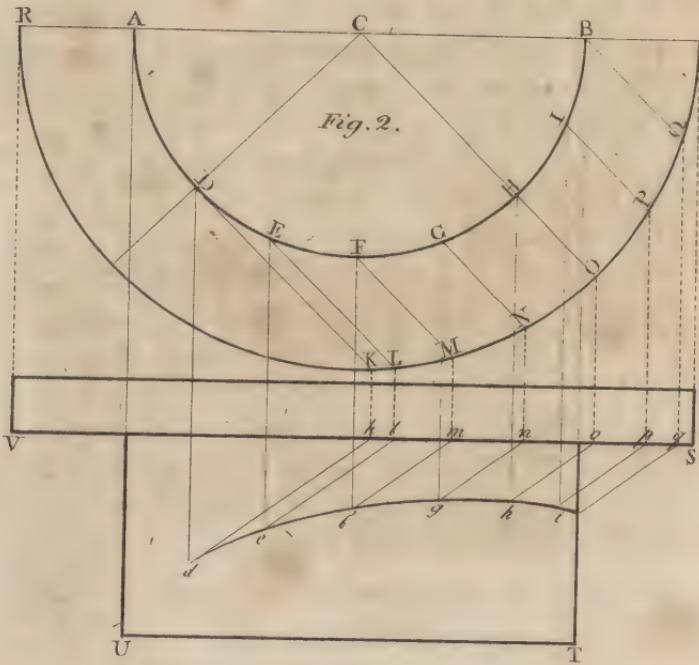
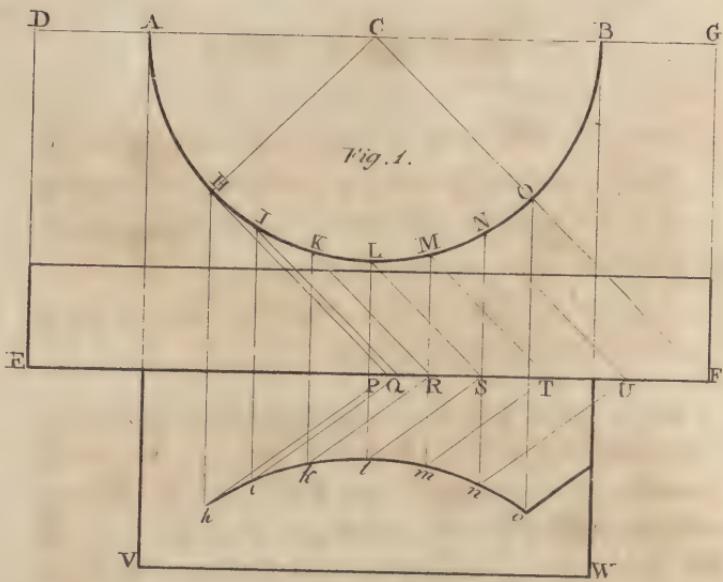
Drawn by P. Nicholson.

Engraved by John Taylor.





## SHADOWS



## PROBLEM VI.

Given the ichnography and elevation of a cylinder, having a square projection or abacus, and the seat of the sun on the ichnography, also its seat on the elevation; to find the shadow of the abacus, also the line of light and shade on the cylinder.

Let A H I K L M N O B be the ichnography of the cylinder; D E F G be that of the abacus; V W the elevation of the cylinder; and E F the elevation of the abacus; let C F be the seat of one of the sun's rays on the ichnography, passing through the centre C; draw F o, making an angle with E F, equal to the angle which the seat of any of the rays make with E F; through C, draw C H, perpendicular to C O, cutting the ichnography at H; between the points H and O, take any points I, K, L, M, and N; then through the points H, I, K, L, M, N, draw lines B B, I Q, K R, L S, M T, N U, parallel to C F, cutting H P at P, Q, R, S, T, U; through the points, P, Q, R, S, T, U, draw lines parallel to F o; also through the points B, I, K, L, M, N, O, draw lines H h, I i, K k, L l, M m, N n, O o, parallel to the sides of the cylinder, cutting P h, Q i, R k, S l, T m, U n, F o, at the points h, i, k, l, m, n, o; through these points draw a curve, and it will be the shadow of the abacus; H h will be the line of shade, and O o the line of light; this appears clearly by the shadowed plate 86.

## PROBLEM VII.

FIG. 2.

Given the ichnography and elevation of a cylinder, having a circular projection over it; the seat of the sun on the ichnography; also its seat on the elevation being given; to find the shadow of the projection on the cylinder; also the line of light and shade.

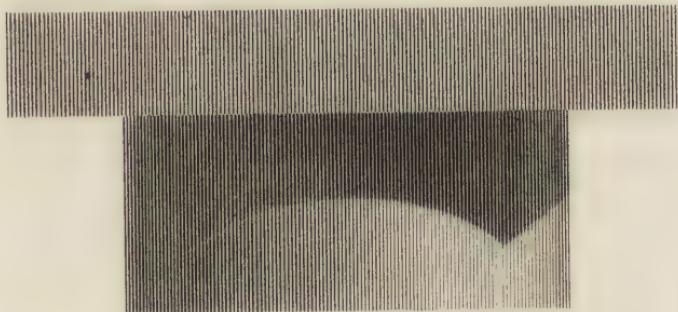
Let  $A D E F G H I B$  be the ichnography of the cylinder, and  $R K L M N O P Q$ , the ichnography of the projection; let  $U T$  be the elevation of the cylinder, and  $V S$  that of the projection: also let  $C O$  be the seat of the sun's rays, passing through the centre  $C$  of the ichnography, and  $o h$ , the seat of the sun on the elevation.

Through  $C$ , draw  $C D$  perpendicular, cutting the circumference of the inner circle at  $D$ , take any point  $E, F, G, I, B$ , in the circumference, draw the lines  $D K, E L, F M, G N, H O, I P$ , and  $B O$ , parallel to  $C H$ , cutting the outward circle at the points  $K, L, M, N, O, P, Q$ ; from the points  $D, E, F, G, H, I$ , draw the lines  $D d, E e, F f, G g, H h, I i$ ; also through the points  $K, L, M, N, O, P, Q$ , draw the lines  $K k, L l, M m, N n, O o, P p, Q q$ , cutting  $V S$  at the points  $k, l, m, n, o, p, q$ ; through the points  $k, l, m, n, o, p, q$ , draw  $k d, l e, m f, n g, o h, p i$ , parallel to  $o h$ , cutting the  $D d, E e, F f, G g, H h, I i$ , at the points,  $d, e, f, g, h, i$ : a curve being traced through these points, will be the representation of the shadow upon the cylinder:  $D d$  will be the line of shade, and  $H$  the line of light.

PROB-

## SHADOWS

*Fig. 1.*



*Fig. 2.*

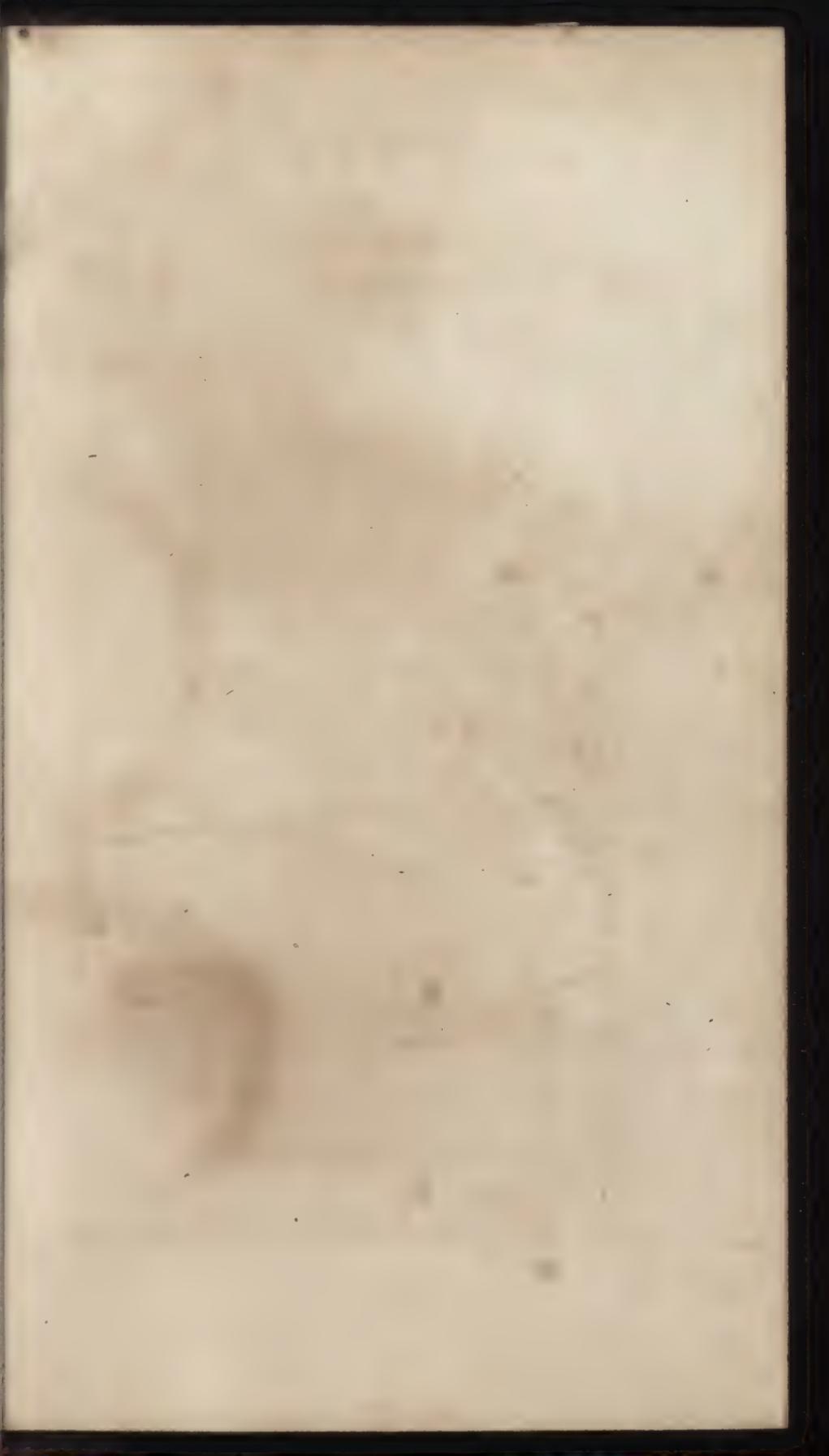


Drawn by P. Nicholson

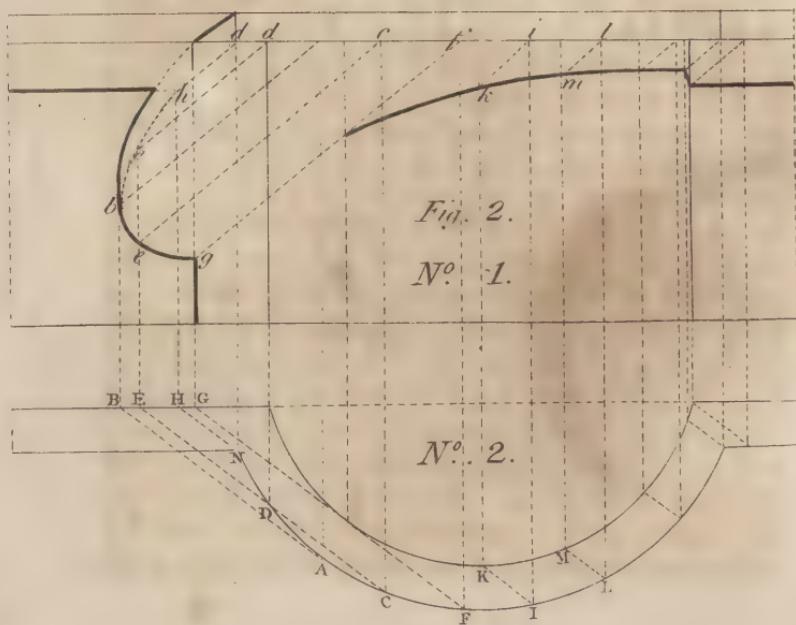
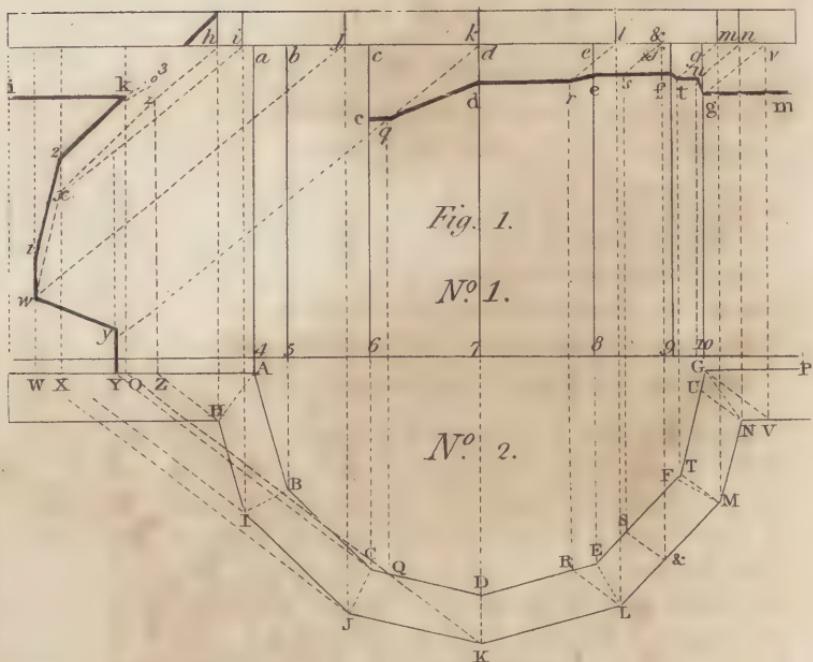
Published as the Act directs by P. Nicholson Feb<sup>Y</sup> 1795

Engraved by W. Lowry





## SHADOWS



## PROBLEM VIII.

## PLATE 87.

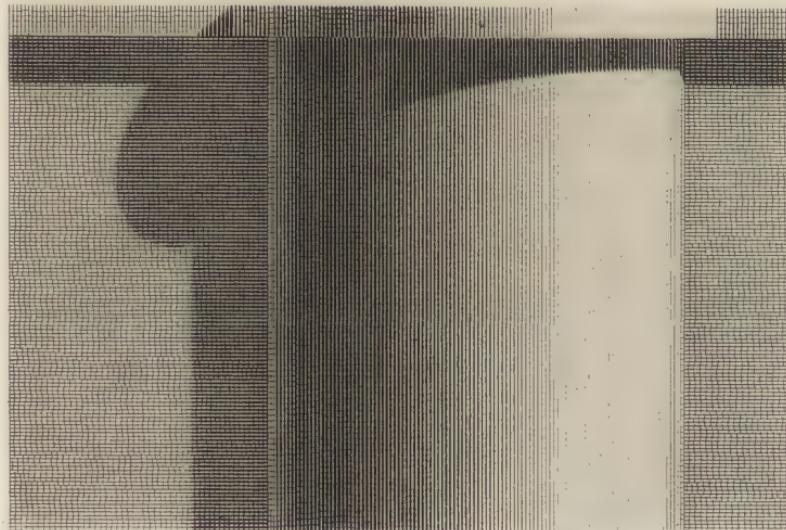
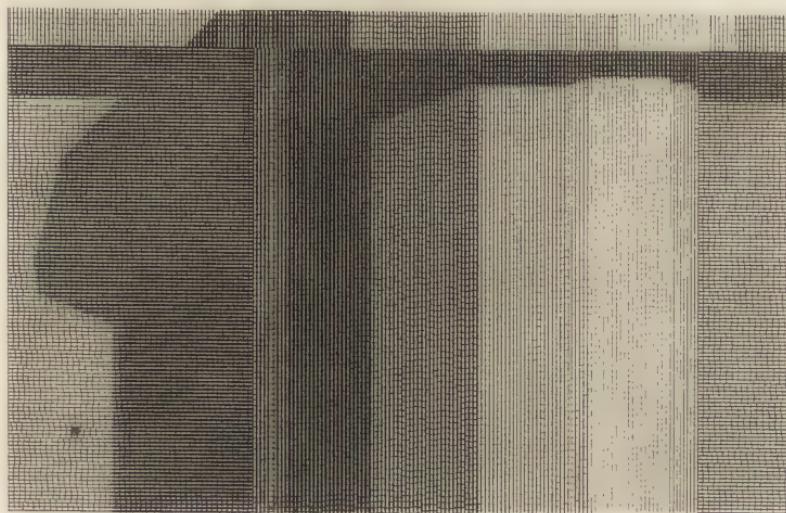
The ichnography and elevation of the prism standing upon a polygonal base, having the projection or cap of the same figure upon it, projecting equally over its sides ; given the seat of the sun's rays on the plane of the ichnography, and also on the elevation ; to project the shadow of the cap on the prism, and also on a plane parallel to the axis of the prism.

Let A B C D E F G be the ichnography of the prism, H I J K L M N the ichnography of the cap WA, and G P, parts of the ichnography of the plane on each side of the prism, J W be the projection of a ray on the ichnography, and  $j w$  one on the elevation. From all the points H, I, J, K, L, M, N, in the ichnography of the angles of the cap, draw the lines H h, I i, J j, K k, L l, M m, N n, perpendicular to W P, to cut the under side of the cap at  $h, i, j, k, l, m, n$ . Draw the lines A 4 a, B 5 b, C 6 c, D 7 d, E 8 e, F 9 f, G 10 g, perpendicular to W P, cutting the bottom of the prism at 4, 5, 6, 7, 8, 9, 10, then the lines 4 a, 5 b, 6 c, 7 d, 8 e, 9 f, 10 g, represent the angles on the elevation. Draw the lines H Z, I X, J W, K Q, L R, M T, N U, parallel to J W, cutting the ichnography of the plane at the points Z, X, W, and the ichnography of the prism, at Q, R, T, U. Through the points  $h, i, j, k, l, m, n$ , the projection of the angles of the cap on its under edges, and parallel to  $j w$ , draw the projections of the rays  $h z, i x, j w, k q, l r, m t, n u$ . Draw the perpendiculars Z z, X x, W w, Q q, R r, T t, U u, then the points z, x, w, are the projections of the angles of the cap on the plane, and q, r, t, u, the projection of the other angles on the elevation of the prism. Draw C Y to touch the prism at C, and draw Y y perpendicular to W P, then Y y will be the termination of the shadow of the body of the

prism on the wall. Then because that the point  $q$  is in the plane  $6\ c\ d\ 7$  the point  $r$  in the plane  $7\ d\ e\ 8$ , and the points  $t$  and  $u$ , in the plane  $9\ f\ g\ 10$ , and the lines  $j\ k$ ,  $k\ l$ ,  $m\ n$ , parallel to these planes; the lines  $j\ k$ ,  $k\ l$ ,  $m\ n$ , will make parallel shadows; therefore draw  $q\ c$ ,  $r\ d$ ,  $t\ u$ , parallel to  $j\ k$ ,  $k\ l$ , or  $m\ n$  to cut  $c\ 6$  in  $c$ ,  $k\ 7$  in  $d$ , and  $G\ g$  at  $u$ : join  $d\ q$ ; then to find the depth of the shadow of  $l\ m$  on the elevation, draw  $\&\ s$  on the ichnography parallel to  $J\ W$ ; draw  $\&\ s$  perpendicular to  $W\ P$ , likewise  $\&\ s$ , parallel to the ray on the elevation, and  $S\ s$  parallel to the axis of the prism; then  $s$  is the depth of the shadow: but as the projections of the extremities of  $l\ m$  fall upon the adjacent planes at  $r$  and  $t$ , draw  $e\ f$  through  $s$ , cutting  $8\ e$  at  $e$ , and  $9\ f$  at  $f$ ; then join  $e\ r$ ,  $f\ t$ . Lastly, draw  $G\ V$  on the ichnography parallel to  $W\ J$ , cutting the projection of the cap at  $V$ : draw  $V\ v$  perpendicular, cutting the cap at  $v$ , and draw  $v\ g$  parallel to the rays on the elevation, and join  $u\ g$ ; then  $c\ q\ d$ ,  $r\ e\ f\ t\ u\ g$  will be the shadow of the cap on the elevation; but as the shadow of the parts of the abacus  $H\ I$ ,  $I\ J$ , will be from the top of the cap: make  $w\ 1$ ,  $x\ 2$ ,  $z\ 3$ , parallel to the angles of the prism on the elevation, and equal to the thickness of the cap. Join  $1, 2; 2, 3$ : then will  $3, 2, 1\ w\ y$  be the shadow of the abacus on the wall. Through  $g$  draw  $g\ m$  and  $k\ i$  in the same straight line with  $g\ m$ , cutting  $3, 2$  at  $k$ ; then  $i\ k\ 2, l\ w\ y$  is the complete shadow of the cap on the plane.

Much after the same manner may the shadow of a cylinder be found, having a circular projection over it, as is shown at fig. 2; and also the shadow of the cylinder and projection may be found on a plane parallel to the axis of the cylinder, having the same thing given as before; but for more easy inspection, letters are placed on the ichnography and elevation, representing the corresponding parts of each other: that is, capital letters are placed on the ichnography, and small letters of the same name on the elevation, representing those of the same name on the ichnography.

SHADOWS.



P. Nicholson del.

W. Lowry, sculp.



## PROBLEM IX.

Given the seat and altitude of the sun on any plane, also the seat and altitude of a line to the same plane ; to determine the shadow of the line upon that plane.

Let  $K L$  be the seat of the line upon the plane, and  $H G I$  the angle which the line makes with its seat ; make the angle  $M K L$  equal to the angle  $H G I$  ; through  $L$  draw  $L M$  perpendicular to  $K L$ , cutting  $K M$  at  $M$  ; also through  $L$  draw  $L O$  parallel to the seat of the sun ; again, through  $L$  draw  $L N$  perpendicular to  $L O$ , and make  $L N$  equal to  $L M$  ; then upon the right line  $L N$ , and from the point  $N$ , make the angle  $L N O$  equal to the compliment of the angle of the sun's inclination to a right angle ; produce  $N O$ , cutting  $L O$  at  $O$  ; then join the points  $K$  and  $O$ , and the line  $K O$  will be the shadow required.

This problem will be found of great use in finding the shadows upon inclined planes.

*Note.* If the seat and altitude of the sun be given on any other plane, making a given angle with the plane on which the shadow is to be projected ; the sun's altitude and seat may be found by Problem II.

## PROBLEM X.

## PLATE 89.

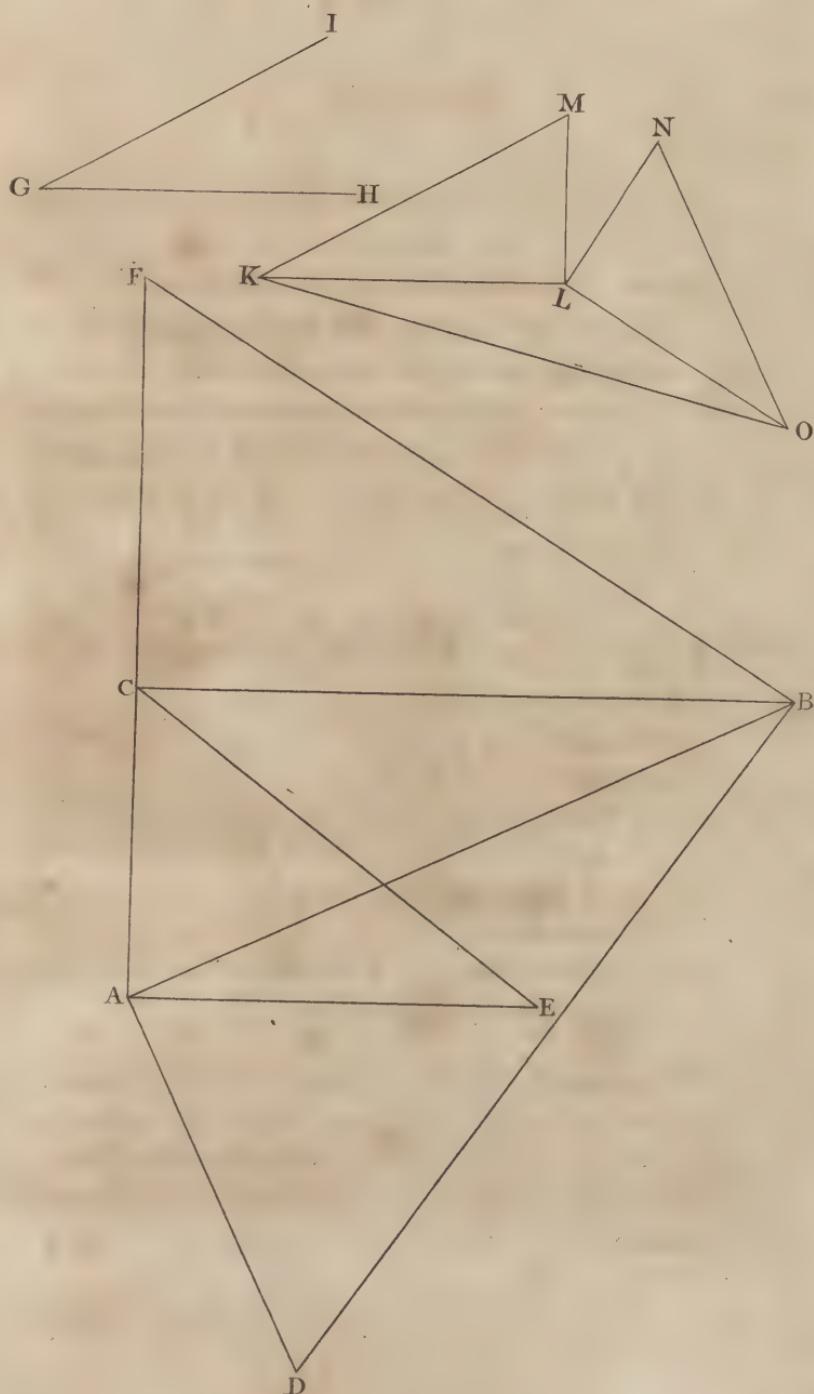
Given the seat and angle of inclination of the sun on the horizon, and the intersection of a plane perpendicular to the horizon ; to determine the angles which a plane of shade made by a right line, parallel both to the horizontal and perpendicular planes, will make with each of the planes.

Let A B be the seat of the sun on the horizon, and A B D the angle of inclination to the seat A B, and let C B be the intersection of a plane perpendicular to the horizon ; take any point C in C B, and from C, draw C A perpendicular to C B, cutting A B at A ; from A, draw A E perpendicular to A C, and A D perpendicular to A B, cutting D B at D ; make A E equal to A D, and join C E, then will the angle B C E be the angle which the plane of shade makes with the perpendicular plane, and the angle A C E the angle which the plane of shade makes with the horizon.

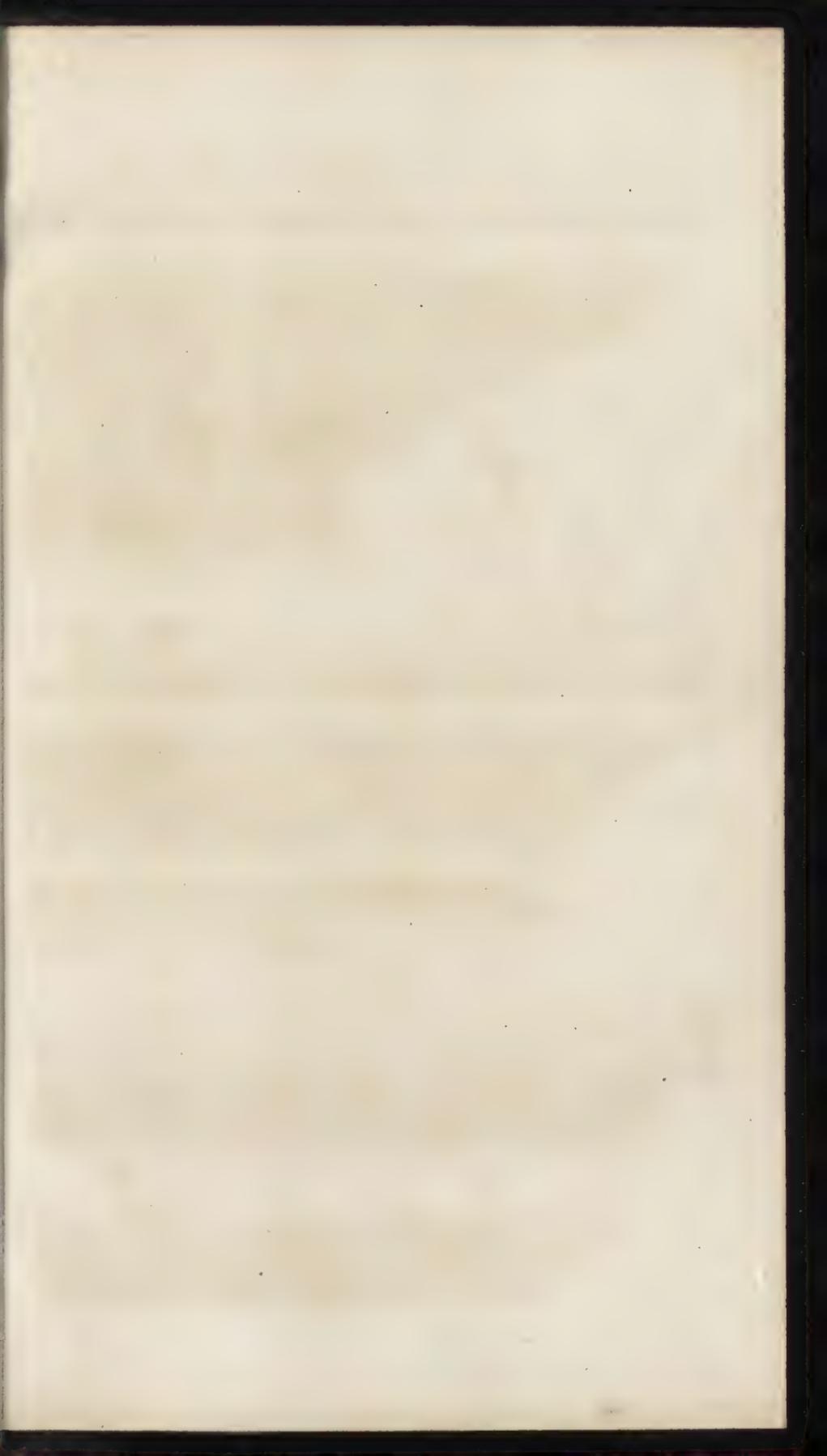
This problem will be very useful in shading mouldings which project from planes that stand perpendicular to the horizon, the sun's altitude and seat being given to the horizon ; as will be shown in the next problem.

PROB-

## SHADOWS

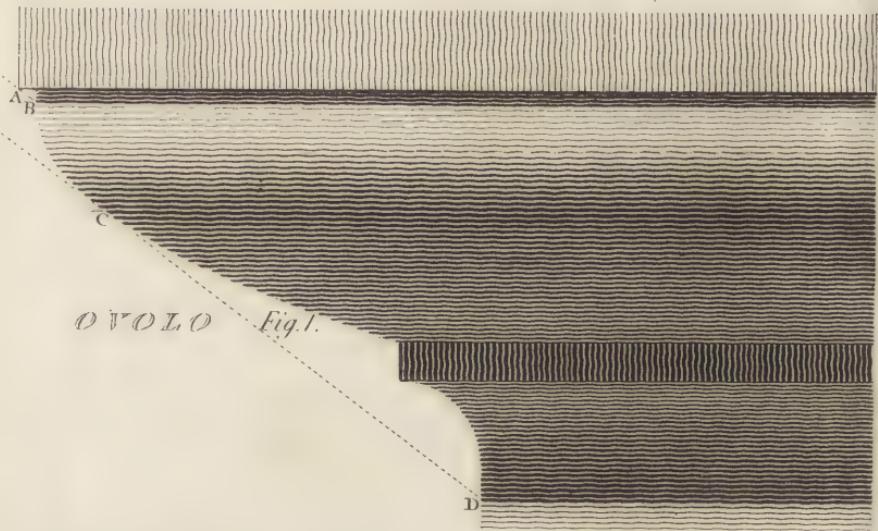




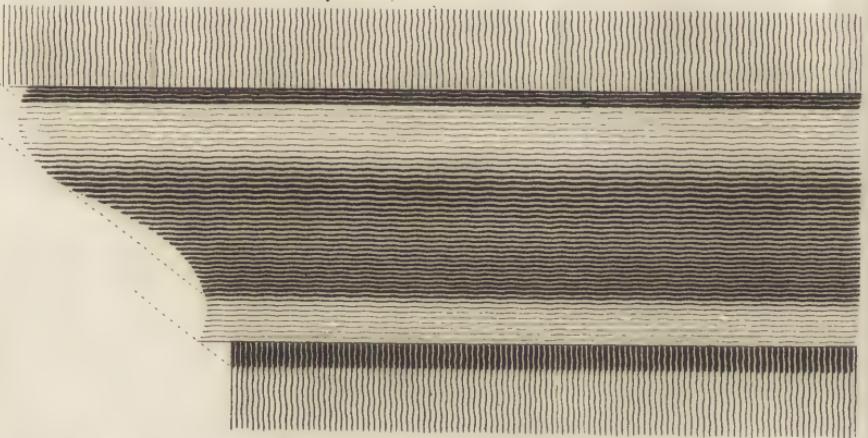


SHADOWS

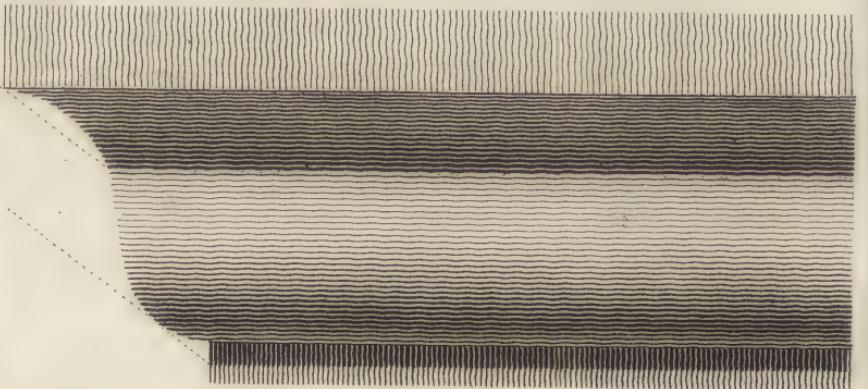
Pl. 9



CIMA REVERSA Fig. 2.



CIMA RECTA Fig. 3.



P. Nicholson del.

W. Lowry sculp.

Published by P. Nicholson & C<sup>o</sup> Oct<sup>r</sup> 1745.

## PROBLEM XI.

## PLATE 90.

A moulding of any kind being given, and the angle which a plane of shade makes with a perpendicular part of the moulding, either being given or found by the last problem, having the sun's altitude and seat on the horizon; to determine the shadow on the moulding.

Let the ovolo, fillets, and hollow, fig. 1. be the given moulding; draw C D parallel to the inclination which the plane of shade makes with the vertical part of the moulding, touching the ovolo at C, and cutting the vertical part below at D; then a line drawn through D perpendicular to the fillet will give the lower edge of the shadow, and an imaginary line supposed to be drawn through C, will give the line of shade; and if a line is drawn through A, the lower edge of the fillet above the ovolo parallel to C D, cutting the ovolo at C, then a line being drawn through B, parallel to the fillet, will give the edge of the shadow from the fillet.

Much in the same manner may the shadow upon the cima reversa and cima recta be found, as shown by the dotted lines.

## SCHOLIUM.

The form or shape of mouldings, in most cases, may be ascertained from the various degrees of light and shade upon them, without observing the profiles; which will appear evident from the following observations.

*Observations on Surfaces, and their Power to reflect Light.*

It has already been observed in the second proposition, that if the sun's rays fall upon a reflecting plane, the angles made by the reflected rays, with perpendiculars at the impinging points, will be equal to the angles made by their corresponding incident rays with the said perpendiculars; so that the rays in this case will have only one direction after reflection: but by experiment we are shown that there is no such thing as a perfect plane; for if a surface is even polished to the greatest degree, yet this polished surface will be but rough and uneven; for if viewed through a microscope having a great magnifying power, the surface will appear quite irregular, and the different parts of the surface will be inclined to any fixed plane, in all manner of directions; and consequently if the sun's rays fall upon such a surface, the rays will not be entirely reflected in the same direction, but a great part of them will be reflected in all manner of directions by the different positions of the surface, by proposition IV. But it may be observed, that the higher any surface can be polished, the nearer it approximates to a plane, and consequently the rays will be more and more reflected in the same direction; therefore there is no surface

face which will reflect the sun's rays entirely in the same direction that is parallel to each other, but a great part of them will be reflected in all manner of directions: it will be also necessary to observe, that the power of reflection will depend very much on the lightness of the colour of materials; for the darker any substance is, the more will the rays of light be absorbed in that substance, and consequently will have a less power to reflect.

White being the lightest of all colours, it will therefore reflect the most rays; and the more any substance inclines to a white, the greater power will that surface have to reflect the rays of light.

*Observations on Mouldings in Shade.*

CASE I.

If the sun's rays fall upon any building; also upon the ground or horizon below the building; and if there are any projectures from the building, such as mouldings or other ornaments, and if any of the parts of those mouldings or ornaments are entirely in shadow by the projecture of something else which prevents the rays from falling upon them, those parts of the mouldings which are in shade will become visible; for, besides a reflection from the ground, there will be a strong reflection from the surface of the building, immediately under the mouldings or ornaments. It has already been observed, that these rays will be reflected in all directions, and consequently a part of them will be reflected upwards on the mouldings above, and therefore will show light and shade on the mouldings, according as the reflected rays fall, more or less perpendicular on their surfaces.

Hence the reason why all perpendicular sides of fillets will be darker in shade than the horizontal sides.

An *ovolo* having a projecture over it, so as to prevent the sun's rays from falling upon it, the reflected rays being more and more inclined from the under edge towards the upper edge, it will therefore be lightest below, and will be gradually darker and darker upwards.

A *cavetto* or *hollow*, immersed in shade, will, for the same reason, be darkest below, and will be continually lighter to the upper edge.

A *cimareversa*, in shade, will be darkest above and below, and lightest in the middle; for this moulding is composed of an *ovolo* above, and a *cavetto* below.

A *cimarecta*, in shade, will be lightest above and below, and darkest in the middle.

These are general rules for shading horizontal mouldings.

### *Observations on vertical Mouldings.*

#### CASE II.

All upright or perpendicular mouldings, in shade, or being in part so, will receive a reflection from those surfaces which are next to them; for they cannot receive a reflection from the contrary side, by reason of their projection, which will prevent the ray that is reflected from that side from falling on them.

Therefore it is plain, in these cases, the forms of mouldings may be known by reflection.

Artists give this rule for shadowing: that is, to shade all mouldings or other ornaments which are in shade, inverse to those on which the sun's rays fall, from the contrary side of the reflected rays. But this rule is not only very uncertain, depending much on the situation of other objects which surround these mouldings or ornaments, but in some cases very erroneous, as in the example of mouldings perpendicular to the horizon: for mouldings in this situation, as has been observed, will receive a reflection from that side which is next to the front of the moulding, if something else does not project to a great distance from that surface from which the reflection comes. If a cylinder or column is attached to a wall in a vertical position, and if it has any projection over it, so as to cause that part under the projection to be in shade quite round the cylinder, there will not only be a reflection from the wall on the contrary side of the cylinder, to the sun upon that side of the cylinder which is next to it, but also from that part of the wall on that side of the cylinder next to the sun, which will make that part of the cylinder which is in shadow lightest at the two sides and darkest in the middle. Something of the same kind may be seen in Ionic columns attached to a wall; where it may be observed, when the sun shines upon one side of them, suppose that side of the column which is on the right hand, then the right hand volute will throw a shadow upon the light side of the column, which shade will be lightest on that edge which is next to the wall and to the luminary, and darkest at that edge next to the middle of the column.

*Observations on Mouldings in Shade, when situate on the Side of an Object which is entirely in Shade, and also the Ground under that Side in Shade.*

CASE III.

In a building where one end or side is entirely in shade, and also a great part of the ground under that end in shade, there will be little or no reflection from the ground upwards, nor from the surface of the building, and consequently little reflection upon the mouldings from below: the only light which they receive is from a kind of scattered or confused rays in the atmosphere, and small reflections from the horizon; and therefore horizontal mouldings, or ornaments, in this situation, which have but small projections over them, will have a contrary effect to mouldings in shadow, situate on the light side of an object.

An *ovolo* placed horizontal, and whose greatest projection is upwards, upon the dark side of an object, will be lightest above, and continually darker and darker to the under edge.

A *cavetto*, having its greatest projection upwards, placed horizontal on the dark side of an object, will be darkest above, and continually lighter and lighter to the under edge.

A *cimareversa*, placed horizontal on the dark side of the object, having its greatest projection upwards, will be darkest in the middle, and lightest above and below.

A *cimarecta*, whose greatest projection is upwards, and placed horizontal, will be darkest above and below, and lightest in the middle.

All horizontal projections on the dark side of an object will condense the shade under them, and consequently will appear more or less dark, according as the projection is more or less.

These are general rules for shading mouldings on the dark side of an object from scattered light; however, there are some exceptions to these rules; that is, when any of these mouldings have a very great projection over them, this projection will hinder the scattered rays from falling upon the mouldings; but as they will receive a small reflection from the horizon below, and therefore the most of the scattered and reflected rays will fall obliquely on the moulding; thus the lightest place of an ovolo will not be exactly on the under edge, but somewhere between the under and upper edge, and will be nearest to either, according as the shadow on the ground is less or more distant from that side of the object, and according as the projection over the moulding is more or less, and also according to the position and distance of other surrounding objects; all these different circumstances combining together, will vary the places of light and shade on horizontal mouldings, which are situate on the dark side of an object.

An horizontal cavetto on the dark side of an object having a projection over it as before, the lightest place will be somewhere between the upper and under edge, as in the ovolo, and both mouldings would have actually  
the

the same appearance if their profiles could not be observed ; when most of the scattered and reflected rays are in a plane, making equal angles with the horizon, and with that side of the object in shade : that is, forty-five degrees with each other ; and consequently mouldings in this situation will be less distinct than mouldings in shade, on that side of the object which the sun shines on.

*Further Observations on the Effect that reflected Light will have on Cornices, which have Modillions or Mutules, Denteles, &c. or any other projecting Ornaments of a nature similar to them.*

The reflected light from the ground and from the object being scattered in all directions, it will therefore follow, if there are any projecting parts from mouldings, or cornices, which are in shadow, such as mutules, modillions, denteles, &c. these projecting parts will hinder a great part of the scattered rays from falling in the spaces between them ; and therefore the spaces will be deprived of reflection, and consequently will be much darker than the prominent parts, even if these prominent parts were also in shadow.

For this reason the intervals between mutules, modillions, denteles, &c. are darker than on their fronts ; for every projecture will condense the shadow on each side of it, if recessed on both sides ; therefore the spaces will be lightest in the middle, and darkest nearest to the edges of the mutules, modillions,

lions, denteles, &c.; but to show on which side of the mutules, &c. the greatest shade would fall, according to the place of the luminary, would be almost impossible, as it depends so much on the situation of other objects. But suppose all the surrounding objects in the vicinity to be removed, and the ground and building to be of a light colour, and suppose the rays to proceed from the right to the left hand of the object, and parallel to a vertical plane which is inclined at an angle of forty-five degrees with the elevation of the object; then it is plain, that since the angle of reflection is equal to the angle of incidence, the greatest part of the rays which fall upon the horizon will therefore be reflected from the ground parallel to the vertical plane; and seeing that the vertical plane would be on the right hand of another vertical plane perpendicular to the face of the object, and to the horizon, it will therefore follow, that most of the rays will come from the right hand, and be reflected towards the left on the object; and consequently any projections from cornices, as mutules, &c. which are in shadow, will condense or darken the shade upon the left hand of the projection; and that vertical side of the mutules which is next to the luminary, will be lighter than the other vertical side on the left of the mutule, &c. As to the direction and effect, which most of the reflected rays would take from the face of the object, imagine a plane parallel to the sun's rays, and perpendicular to the face of the building or object; then most of the rays will be reflected from the building or object downwards, parallel to this last mentioned plane; and that part of these rays which are reflected upwards, would take no particular direction to the right or to the left, and therefore would cause no sensible difference upon the vertical sides of the mutules, but would reflect most light upon the horizontal or under sides, &c.

What

What has now been said of mouldings in shade having projections from them, or of the recessed parts of any object, will apply to ornaments in shade which are deeply relieved; for their recessed parts, according to the foregoing observation, will be deprived of reflection by the more prominent parts of them; they will therefore be darkest in their receding parts, and lightest on the prominent parts.

*Observations on the Shades of Projectures  
from Buildings, or from any other plane  
Surface which is made of light Materials.*

If there is any vertical plane, and if a rectangular prism is attached to that plane, having two of its sides parallel to the plane, and consequently the other two sides perpendicular to it; then if the sun-shine on the plane be on either side of the prism, the other opposite side of the prism will cause a shadow to be projected from its edge upon the plane; and if the shadow upon the plane be of no considerable breadth, and if the plane be extended at any considerable distance beyond the shadow, then the lightest part of the plane on which the rays fall, will reflect a great part of the rays toward the prism: but as these reflected rays will not fall upon the shadow, it will therefore be deprived of reflection: but as the side of the prism which projects the shadow is opposed to the reflected rays, that side of the prism will receive a strong reflection, which will cause it to appear much lighter than the shadow it throws on the plane; but if the shadow be projected farther on the

plane

plane, it will diminish the reflecting surface behind the prism, and will also cause the reflecting surface to be at a greater distance from the side of the prism, and consequently will receive less reflection from the plane; and in general the reflection on the prism will be continually diminished, according as the shadow on the plane is increased, till at last there will be no difference between the shadow on the plane, and the side of the prism which projects that shadow; and if the plane be entirely deprived of light, by the extensive breadth of the shadow, the side of the prism will in general be darker than the shadow on the plane: but this will depend very much on the situation of other objects.

A building consisting of light coloured materials, having a break in the front which projects a shadow on the building, at a small distance from the break, will, for the reason before mentioned, be much lighter on the side of the break, than the shadow projected by it on the building behind it; also columns which are attached to a wall, will project a darker shadow on the wall, than any part of the columns which throw the shadow, provided that the shadow is not at any considerable distance from the column; for, according to the above observations, the broader the shadow, the less the column will appear to be relieved from it.

*Observations on the light Side of the Prism,  
and the Effect that a Reflection from the  
Horizon and from the Object will have on  
the Plane behind the Prism.*

The rays of the sun being reflected from the horizon in all directions, the projection of the prism will therefore prevent a part of the reflected rays from proceeding to the plane behind the prism, and consequently the plane would be something darker than the face of the prism, which is parallel to it; but the side of the prism adjoining to the plane will throw a reflection upon the plane, and therefore it would be difficult to perceive the difference between the face of the prism, and the plane parallel behind the prism. As to the difference of light between the side of the prism which is perpendicular to the plane, and the plane, it will very much depend on the situation of the luminary; for if the luminary is in a plane equally inclined to both, there will be nearly the same degree of light on each; for very little difference will arise from the reflection, except the luminary is more inclined to one surface than another; and then that surface will be darker than the other, according to the obliquity of the rays of the sun on that surface.

## PROBLEM I.

Given the ichnography and elevation of a base and capital, and the seat of the sun's rays on the ichnography and on the elevation; to project the shadows caused by the several parts of itself, and the line of shade upon the base.

Imagine the object to be sliced, or cut, by as many planes, parallel to the axes of the columns\*, and to the sun's rays, as may be thought convenient for the purpose: then it is plain, if a ray of light enter any of those planes, that every part of the ray will be in that plane and that the projecting parts upon the edges of these planes will withhold the rays from a part of the edge of the plane; and the lowest point of that part will give the edge or projection of the shadow of the part which throws the shadow: then if a sufficient number of these points are found, a line drawn through them with a steady hand will give the shadow; the line of shade will be found by drawing lines to touch the several sections parallel to the seat of the sun's rays on the elevation; and a line being drawn through the points of contact of the sections will give the line of shade.

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\* It is not absolutely necessary to suppose the plane parallel to the axis of the column, as in this problem; but the sections formed by planes in this position are more easily found than in any other, for which reason I prefer the above position of planes.

Let **H I J K** be the ichnography of the abacus; **H S K** the ichnography of the ovolو; and **M L P** that of the astragal; the lines **G Y, F X, E W, D V, C U, B T, and A S**, are lines drawn parallel to the ichnography, cutting the front **I J** of the abacus, and from the seats on the ichnography and the several seats on the elevation, the shadows may be described, as is shown in the elevation: then lines are drawn to touch the most prominent parts of those sections; and the places where they cut the other parts of the sections, will be the projection of the several points as before; and a line being drawn through these points will give the shadow.

The part *g f e* is the shadow from the abacus, and *d c b* the shadow from the ovolو; thus the point *g* in the elevation is the shadow of *p*; *f* is the shadow of *n*, and *e* the shadow of *m*; and the shadow of the other part of the abacus would be where the dotted curve line is represented: but as the sun shines on the ovolو, in the middle of the abacus, it will throw the shadow lower than the dotted lines. This will be found by drawing lines to touch the several sections, which will give the points **B, C, D**.

*Note.* That the line of shade on the torus might have been found in a very different manner than is shown in this example, may be seen by the circular ring, plate 8S.

Much after the same manner may the shadow and lines of shade be found on the attic base; as is shown in plates 93, and 94.

# SHADOWS

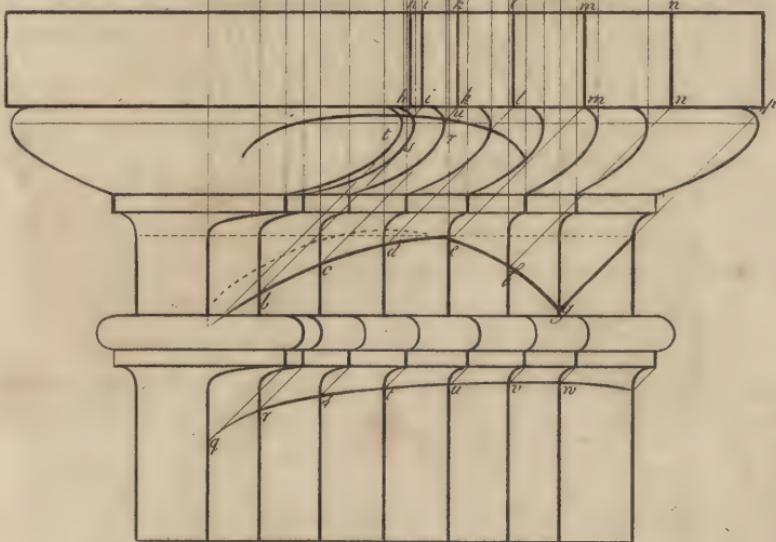
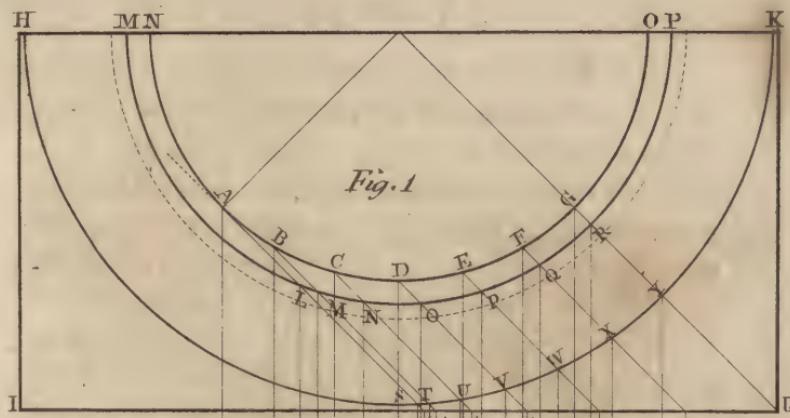
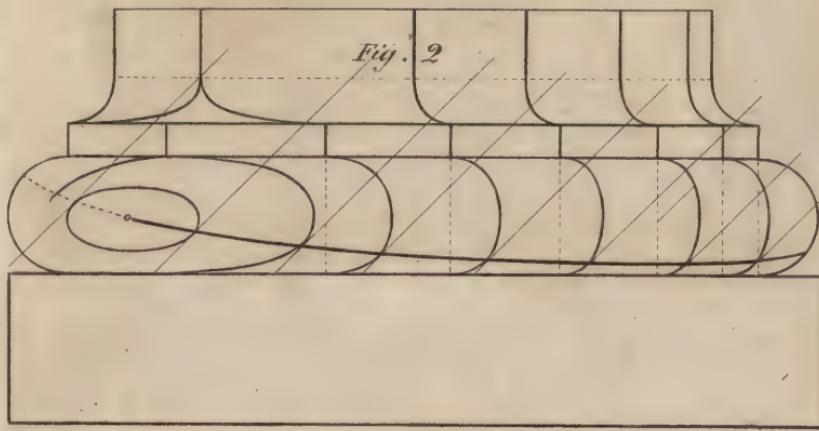
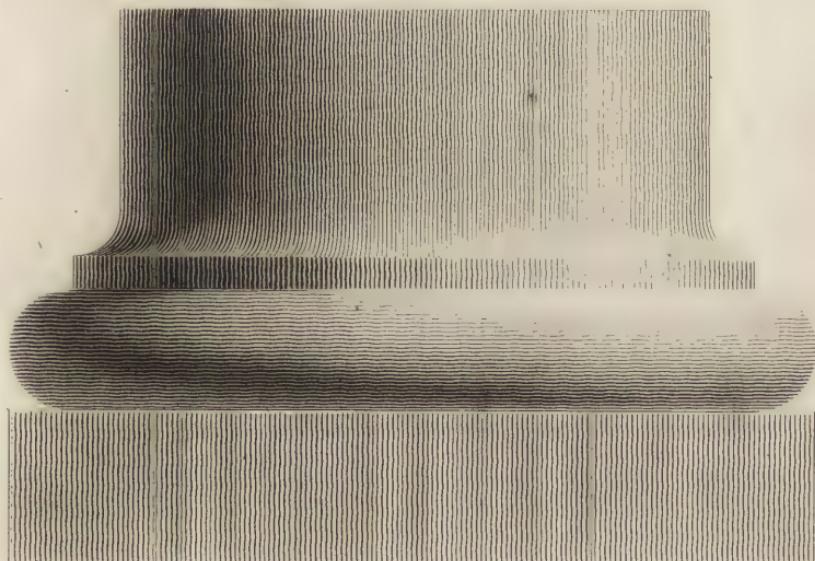
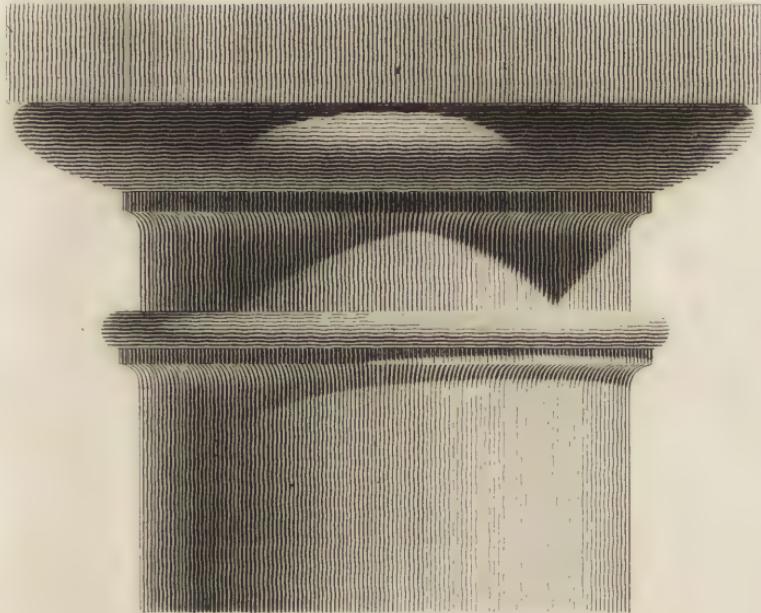


Fig. 2





SHADOWS





## SHADOWS.

Fig. 2.

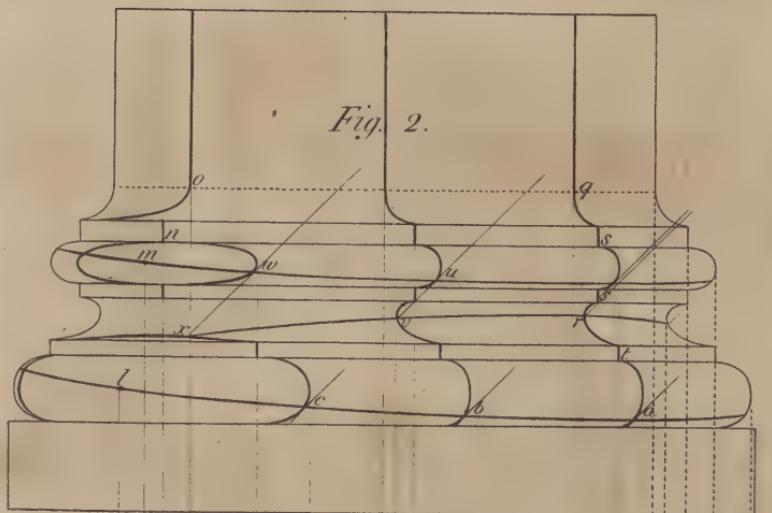
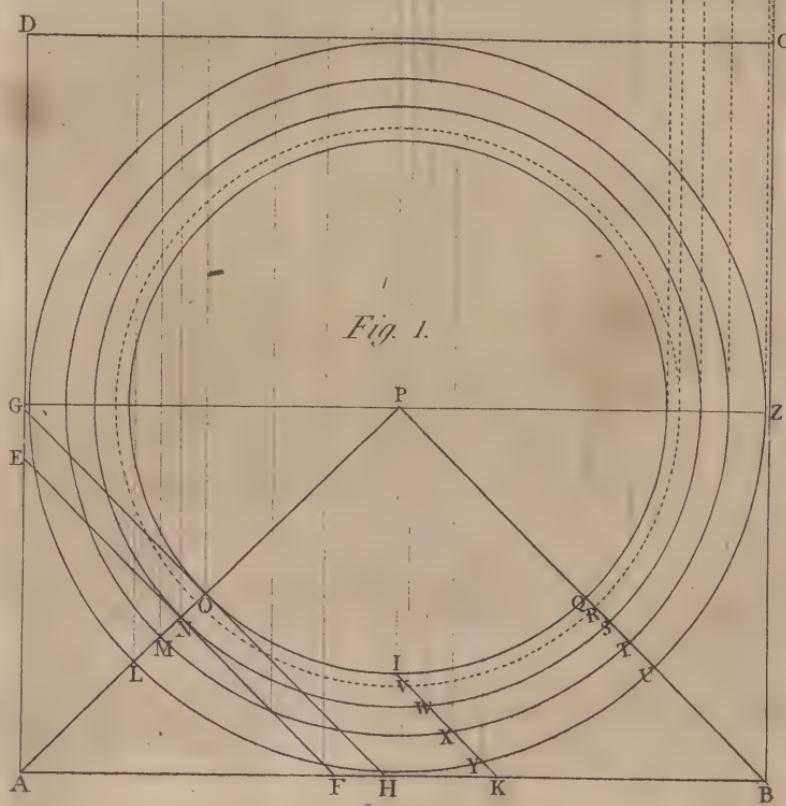


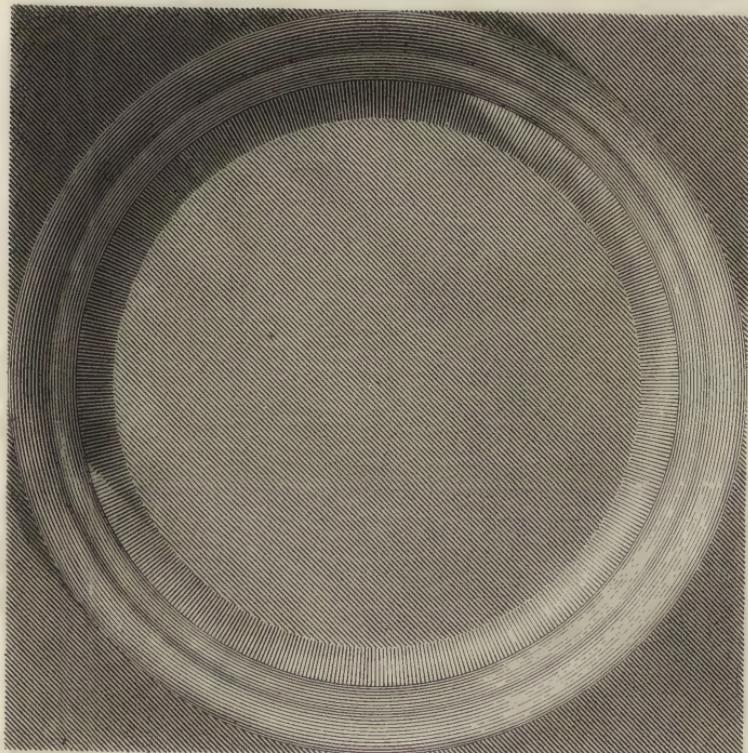
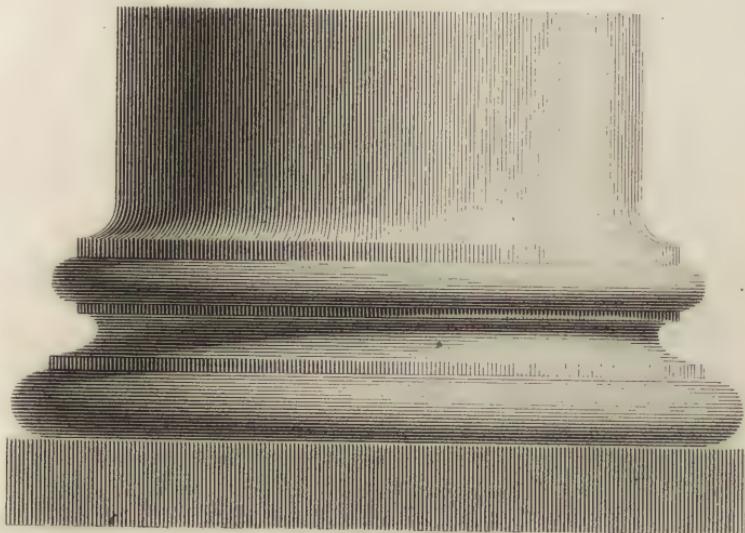
Fig. 1.





SHADOWS

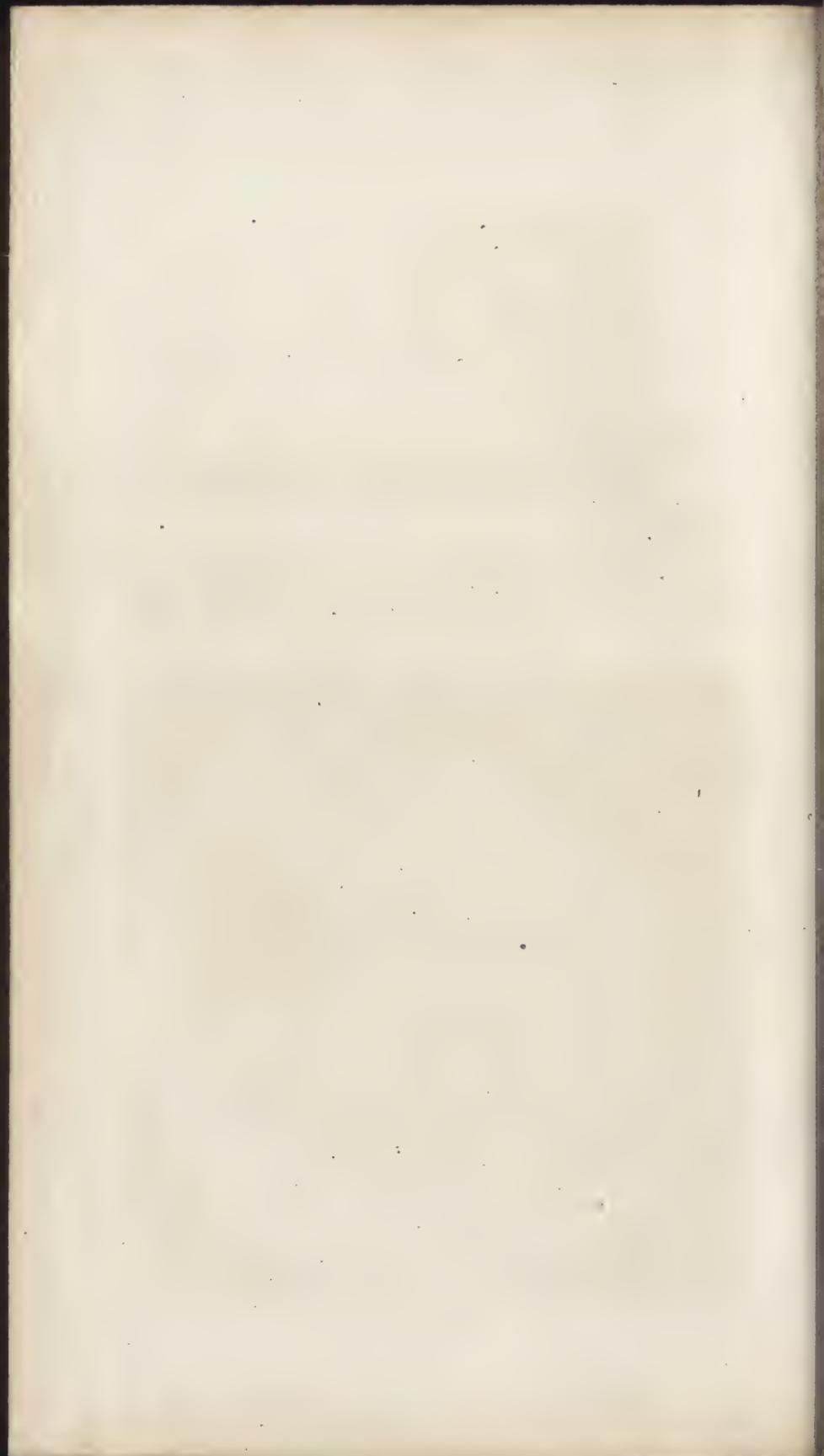
Pl. 94.



P. Nicholson del.

W. Longr. sculp.

London, Published Sept: 1. 1795, by P. Nicholson & C°.



## PROBLEM II.

PLATE 95. FIG. 1.

To find the shadow of a cylindrical recess in a wall, whose axis is perpendicular to the plane of the wall; having the seat of the sun's rays on the ichnography and elevation.

Let fig. 1, be the elevation of the wall, and C D the diameter of the cylindrical recess; and let E F G H be the ichnography; bisect C D at *a*; draw A *a* perpendicular to E H, cutting it at A; through A draw A B parallel to the seat of the sun's rays on the ichnography, cutting F G at B; through B draw B *b* parallel to A *a*; and through *a* draw *a* *b* parallel to the seat on the elevation, cutting B *b* at *b*; then on B as a centre with one half of C D, describe a part of a circle, as is shown by the dark line, and it will be the edge of the shadow.

Much in the same manner may the shadow of a recess, which has a back parallel to the plane of the wall, be found; as is shown at fig. 2.

PROB-

## PROBLEM III.

## PLATE 95. FIG. 3.

To find the shadow of a recess constructed as in fig. 2, when the sides of the ichnography are inclined to the intersection of the two planes of the ichnography and orthography; given, the intersection of a number of planes passing through the luminary perpendicular to the plane of the elevation.

Let  $Z b$ ,  $W Y$ ,  $T V$ , and  $Q S$ , be the intersection of as many planes passing through the sun perpendicular to the elevation, and let  $Q R$  be the projection of one of the sun's rays on that plane; also let  $H I$  be the seat of the sun on the ichnography, cutting the back  $F G$  of the elevation at  $I$ ; from  $I$  draw  $I N$  perpendicular to  $I d$ , the common intersection of the ichnography and orthography; from  $M$  draw  $M N$  parallel to  $Q R$ , cutting  $I N$  at  $N$ ; through  $N$  draw  $N O$  parallel to  $M U$ ; on  $O$  as a centre, and with the distance  $O N$ , describe the arc  $N R$ , cutting the side of the recess at  $R$ ; through the points  $S$ ,  $V$ ,  $Y$ ,  $b$ , draw  $S R$ ,  $V U$ ,  $Y X$ , and  $b a$ , parallel to  $I d$ ; and through the points  $Q$ ,  $T$ ,  $W$ ,  $Z$ , draw the lines  $Q R$ ,  $T V$ ,  $W X$ , and  $Z a$ , cutting the lines  $S R$ ,  $V U$ ,  $Y X$ , and  $B a$ , at the points  $U$ ,  $X$ ,  $a$ ; then through the points  $R$ ,  $U$ ,  $X$ ,  $a$ , draw the curve  $R U X a$ , and the line  $I N R U X a$  will be the edge of the shadow required.

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# SHADOWS

Fig. 1.

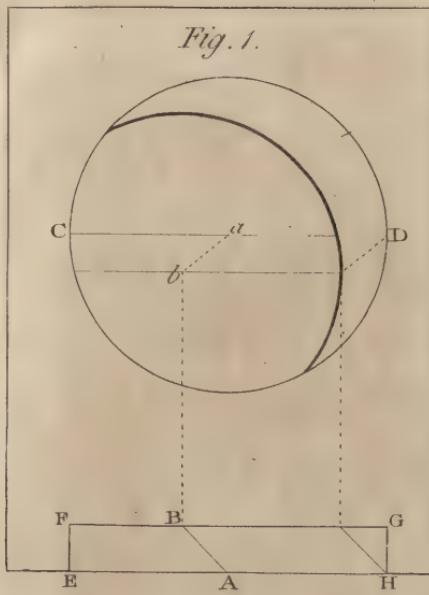


Fig. 2.

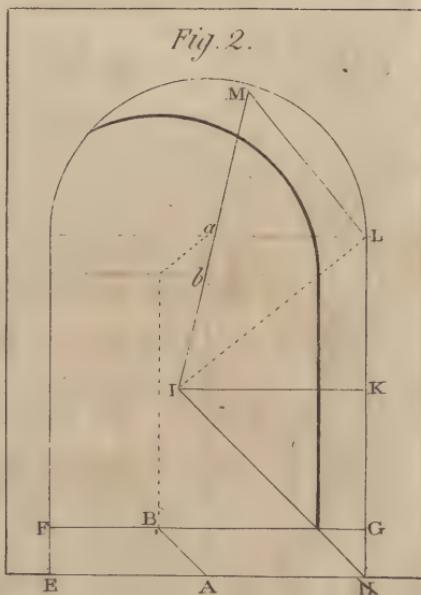


Fig. 3.

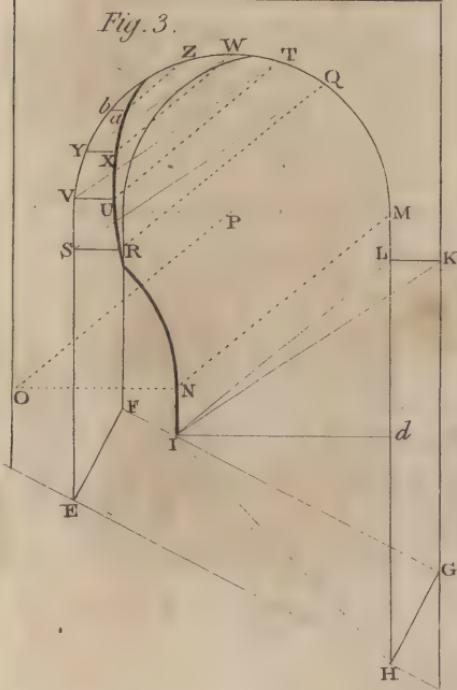
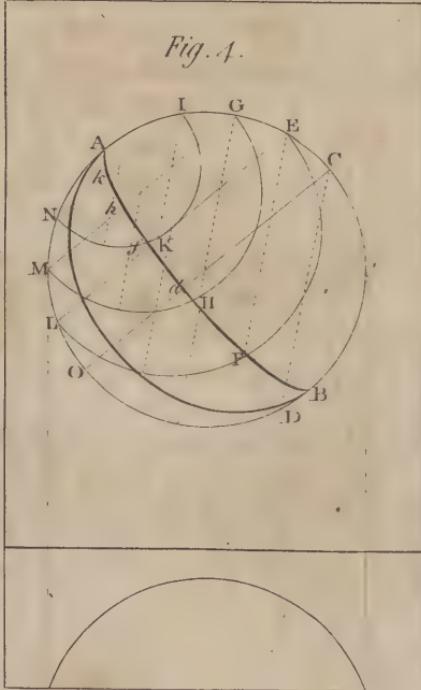


Fig. 4.

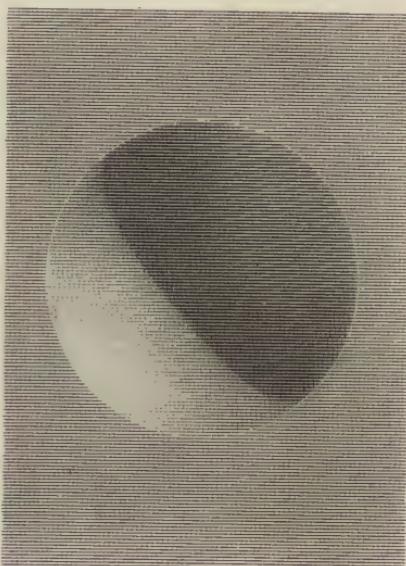
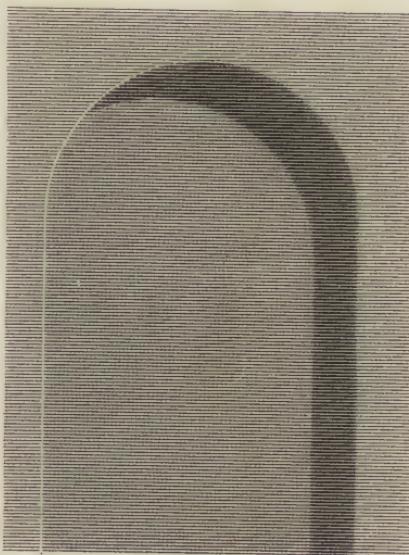
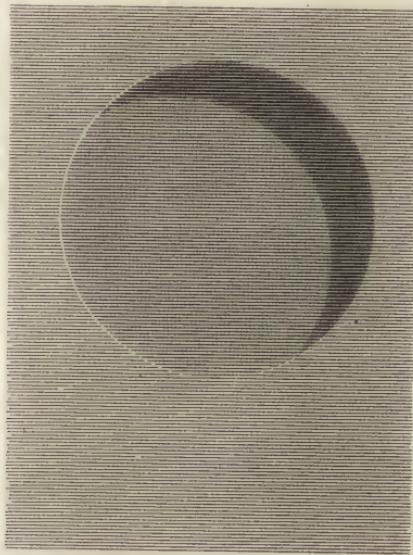


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## SHADOWS

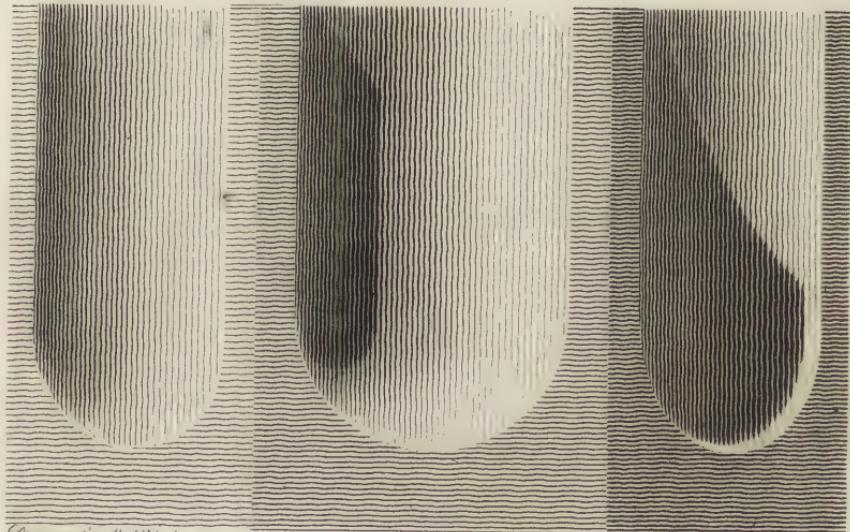
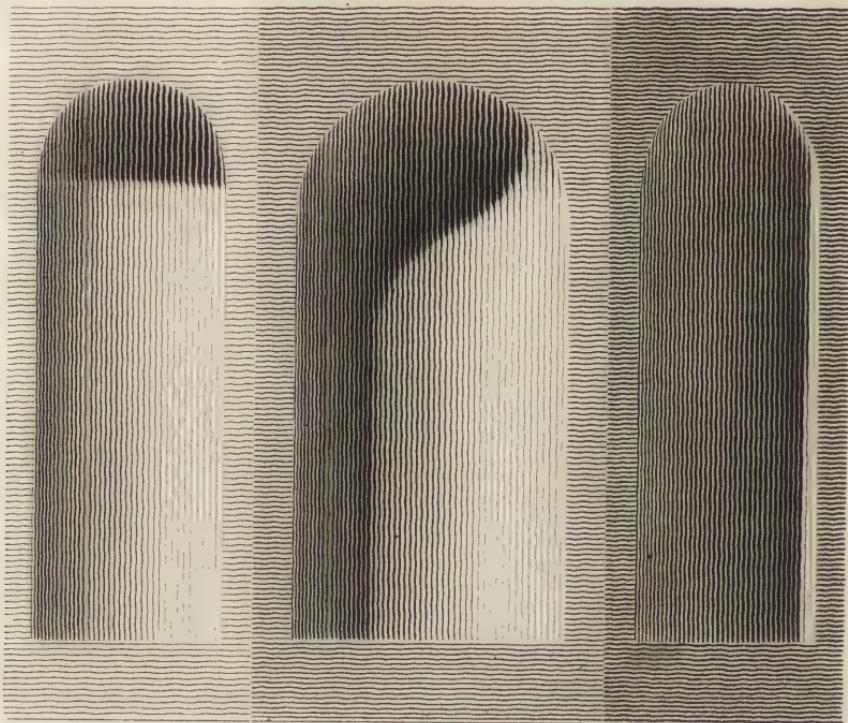


P. Nicholson del.

W. Lewes F. R. S. engrp



## SHADOWS.



Drawn by P. Nicholson.

Engraved by W. Lewry.

London, Published Feb<sup>r</sup> 1790, by P. Nicholson & C<sup>o</sup>



## PROBLEM IV.

PLATE 95. FIG. 4.

To find the shadow of a hemisphere niche; given, the seat and altitude of the sun's rays on the elevation.

Let  $I N$ ,  $G M$ ,  $E L$ , and  $C O$ , be lines parallel to the seat of the sun's rays; and on these lines, as diameters, describe semicircles  $I K N$ ,  $G H M$ , and  $E F L$ ; draw the line  $A B$ , bisecting  $O C$ ; from any of the points  $C$ ,  $E$ ,  $G$ ,  $I$ , as  $C$ , make the angle  $O C D$  equal to the sun's altitude, cutting the side of the niche at  $D$ ; through the other points  $E$ ,  $G$ ,  $I$ , draw  $E F$ ,  $G H$ , and  $I K$ , parallel to  $C D$ , cutting the semicircles  $E F G$ ,  $G H M$ , and  $I K N$ , at the points  $F$ ,  $H$ ,  $K$ ; through the points  $D$ ,  $F$ ,  $H$ ,  $K$ , draw lines  $D d$ ,  $F f$ ,  $H h$ , and  $H k$ , perpendicular to the diameters, cutting them at the points  $d$ ,  $f$ ,  $h$ ,  $k$ ; and through the points  $A$ ,  $k$ ,  $h$ ,  $f$ ,  $d$ , draw a curve, which will be the edge of one half of the shadow, from which the other half may be drawn, as is shown by the figure; and this will give the shadow complete.



In order to show the application of the principles of shadows to enter buildings, the following plates will be found useful in illustrating them. It will be unnecessary here to show the projection of every shadow, which will be only repeating the same methods described in the fore-

foregoing problems, with which it is supposed that the reader is already acquainted.

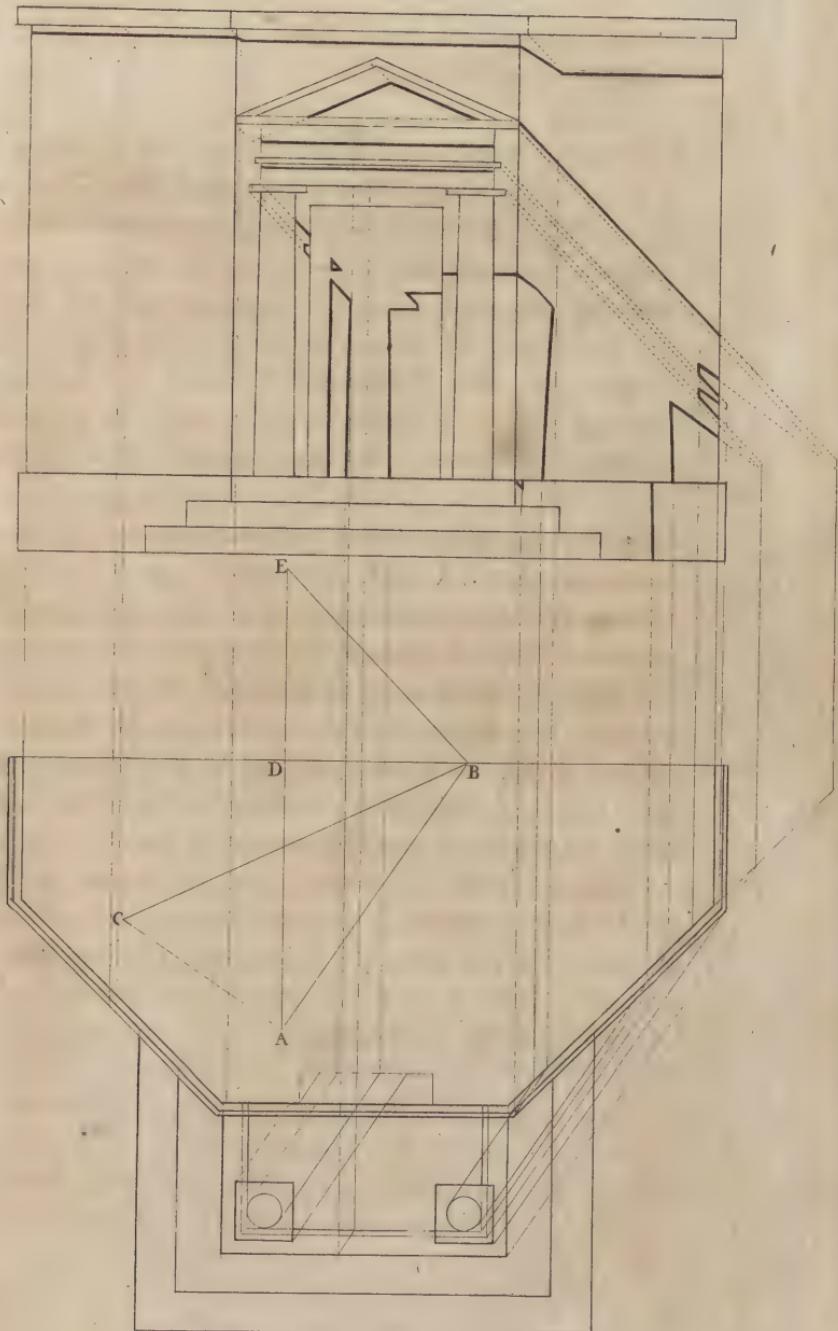
From what has been said in the projection of shadows, the same principles will apply to finding the shadows of the niches, plate 97, projecting the shadows of the columns on the door and on the wall of the octagon tower, plate 98; the shadows of the upper part of the same building, plate 99; these two are shown entire by the finished plate 100. Likewise the shadow of a building having a portico, as is shown by plate 102; the manner of projecting of the shadows being shown in plate 101.

In finding the shadows of the objects as in the foregoing plates, I have in general supposed the luminary to be in a plane, making an angle of forty-five degrees with the horizon and with the object; and also in a vertical plane, making an angle of forty-five degrees with the face of the object; in order that the shadows of vertical and horizontal projectures may throw shadows equal to their projectures, from which we may judge the projecture of different parts of buildings from the shadows, even if the soffit of these projectures could not be seen; however, I have not always adhered to this rule, as may be seen by the octagon tower, plate 100, in order to show the shadow of the columns and the roof upon the third side.

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# SHADOWS

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## SHADOWS

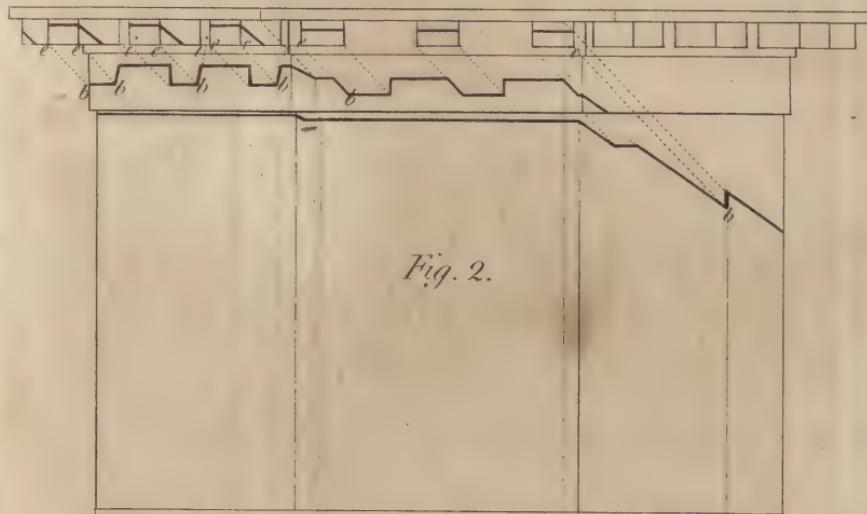


Fig. 2.

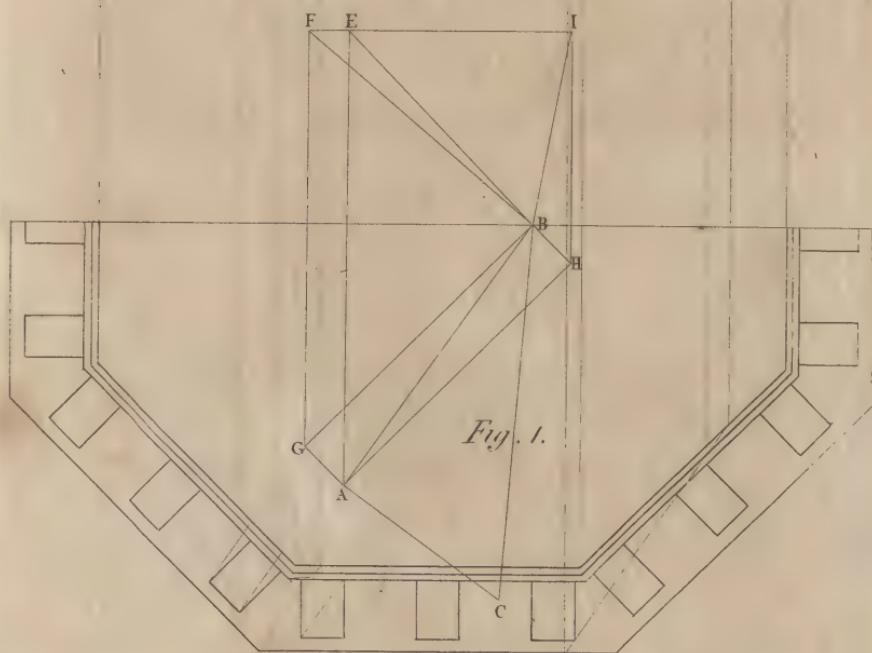
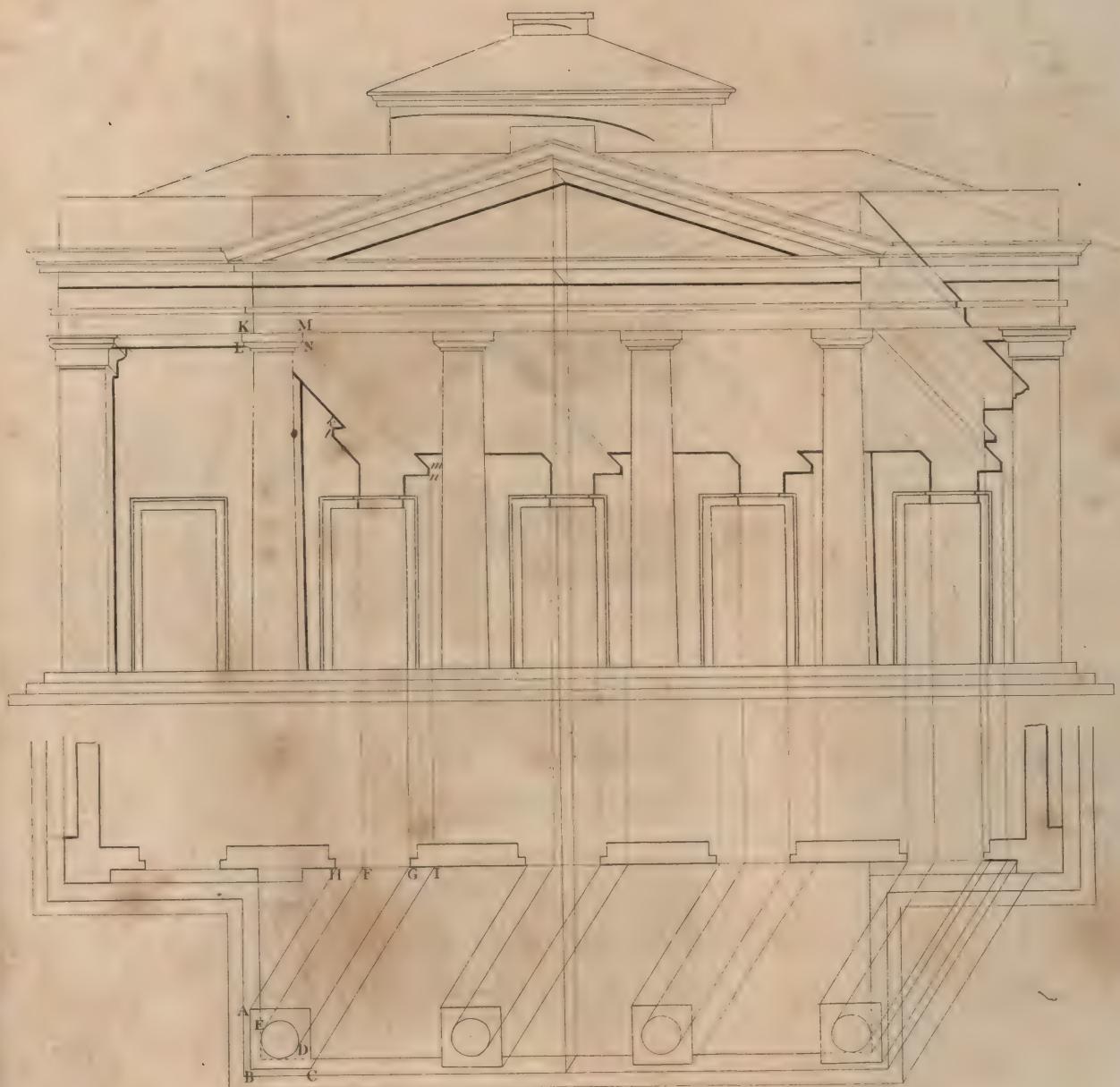


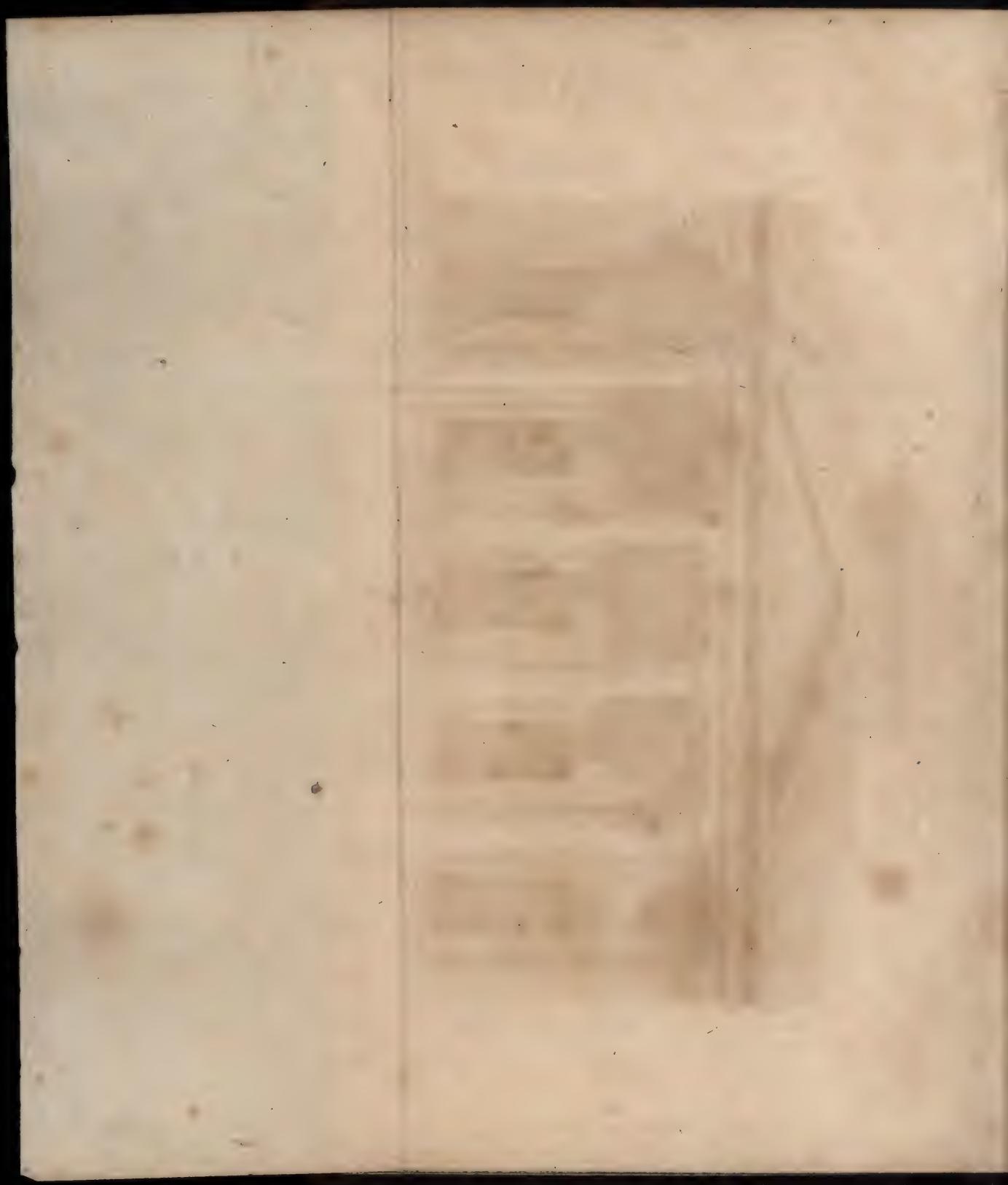
Fig. 1.



SHADOWS



*P. Nicholson del.*



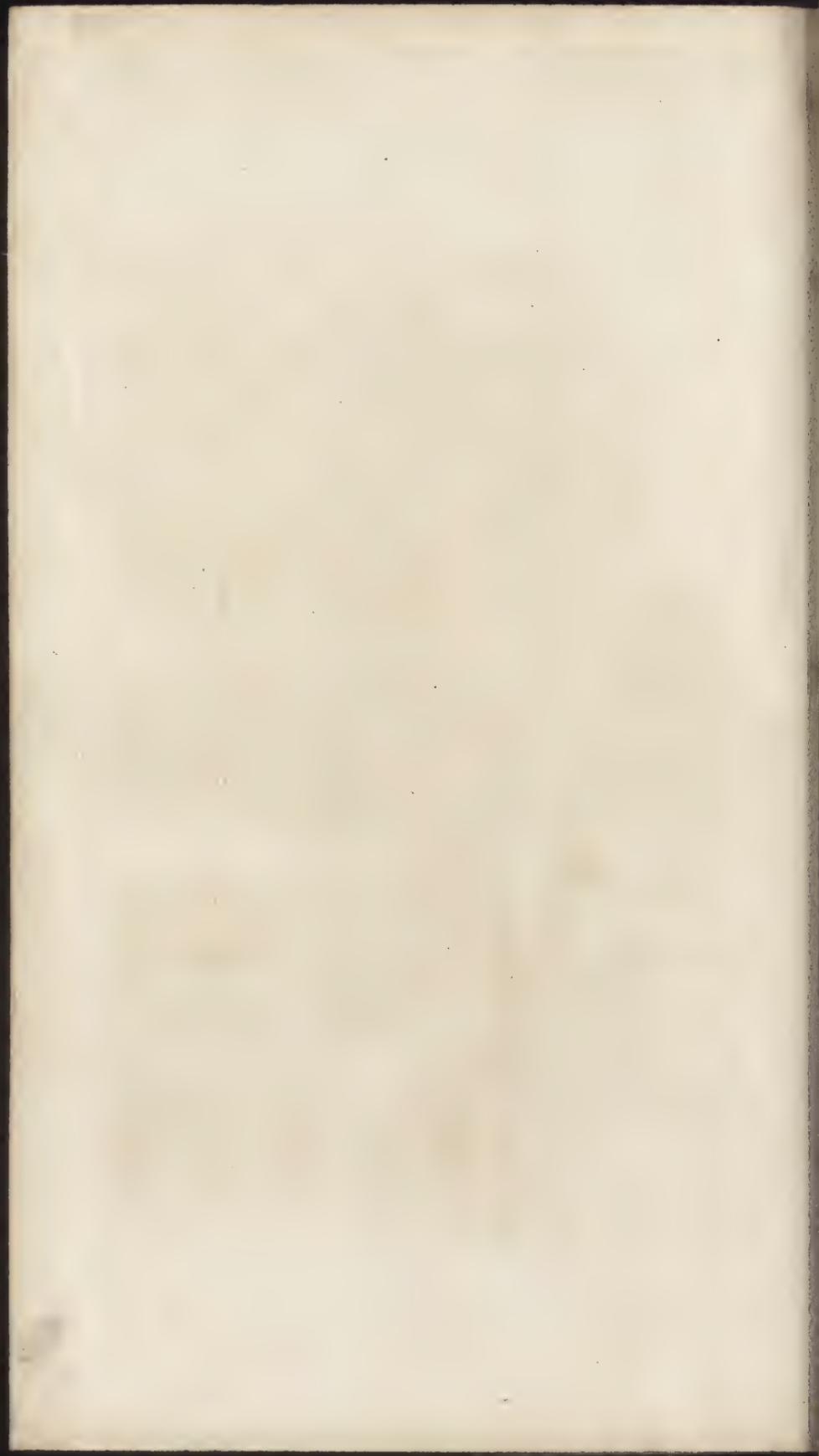
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Published by P. Nicholson, No. 12, Old Bond-street, 1795.

P. Nicholson, M.A.

W. Loney, J. M. S.

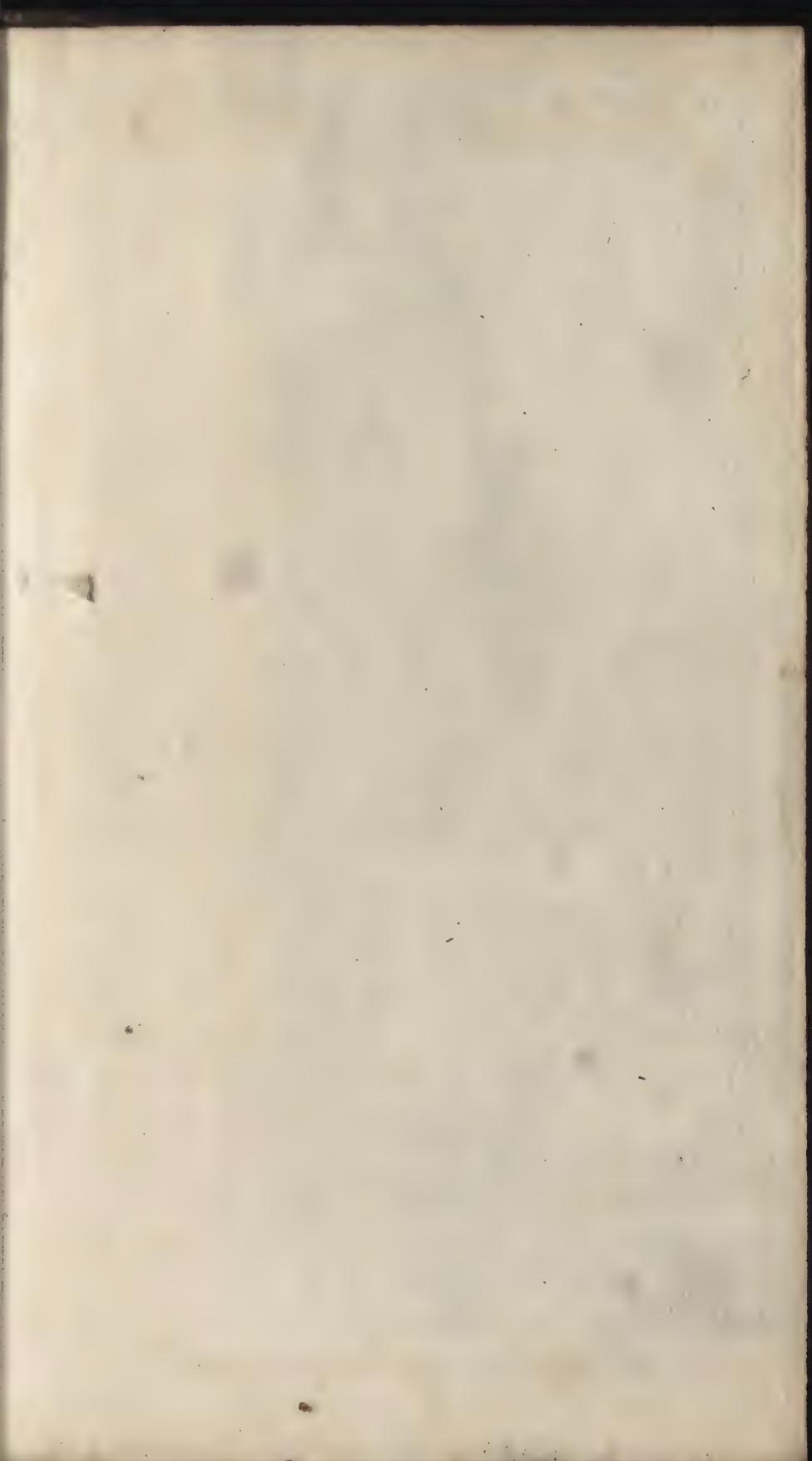


From what has been said on this subject, many practical and useful rules in shadowing may be deduced, but as I have far exceeded the bounds first assigned for this part, I shall therefore end with observing, that from a consideration of the foregoing examples, the shadows of all objects, however complicated, may be found, as every object may be considered as compounded of prisms, cylinders, spheres, and annuluses; therefore each part being projected separately, according to the rules for its particular kind of figure, the projection of all the shadows in that object will be completed.

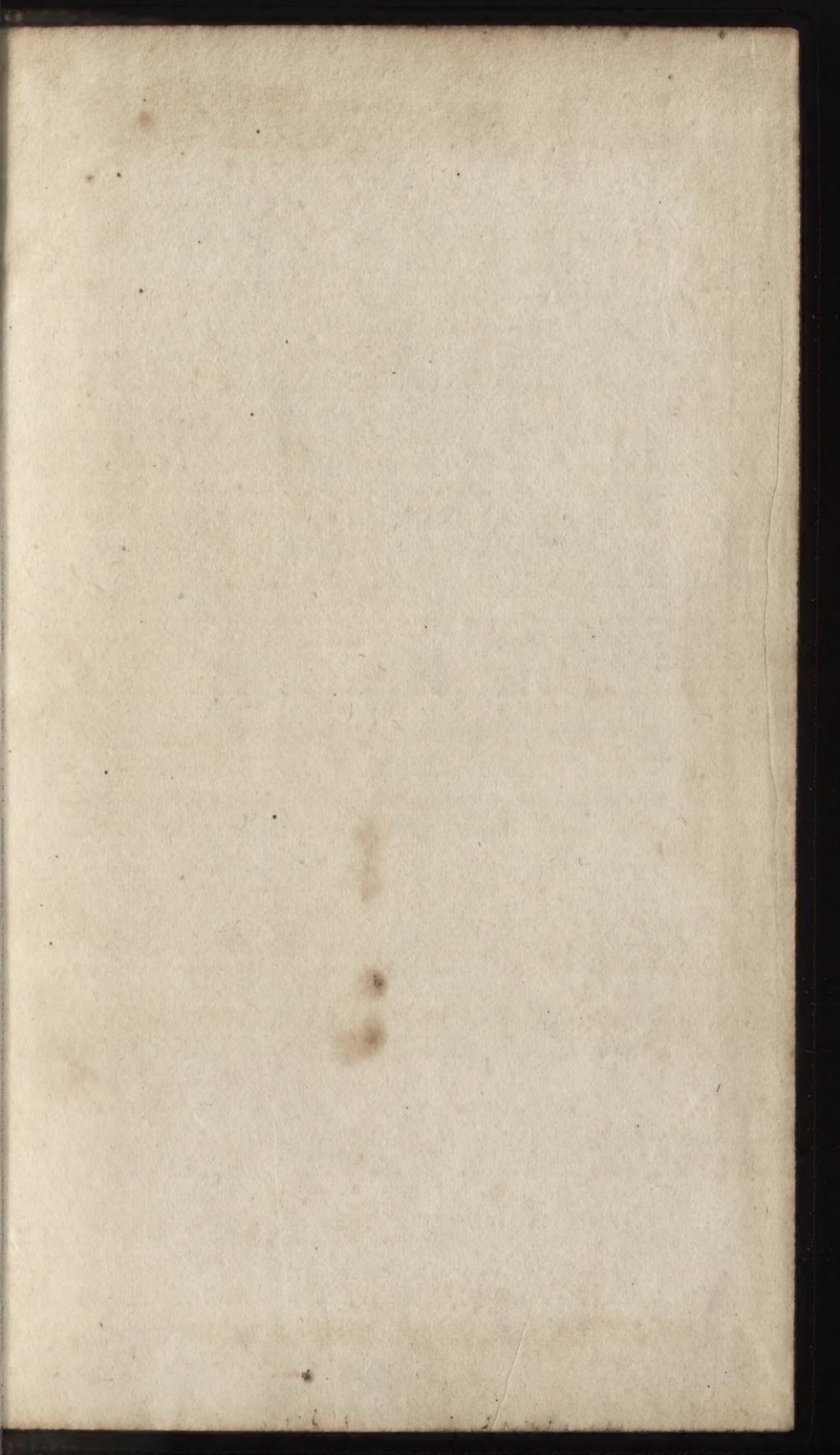
Although I have given correct methods for shadowing, I have no reason to think that the artist will always be at the trouble to project his shadows, for as drawings in general are shadowed to an angle of forty-five degrees, and as I have made choice of that angle, he will therefore still find these examples to be his only guide in practice, as all the forms will be sufficiently near when copied by the eye, and drawn by the hand of a judicious artist.

END OF THE SECOND VOLUME.











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